On the Impulsive Nature of Interchannel Interference in Digital Communication Systems

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***Abstract*— Impulsiveness, or a high degree of peakedness, of interchannel interference in digital communication systems typically results from the non-smooth nature of any physically realizable modulation scheme designed to transmit a discrete (discontinuous) message. Even modulation schemes painstakingly designed to be 'smooth' are not. The non-smoothness of the modulation can be caused by a variety of hardware non-idealities and, more fundamentally, by the very nature of any modulation scheme for digital communications. In order to transmit a discrete message, such a scheme must be causal and piecewise, and cannot be smooth, or infinitely differentiable.**

Recursive differentiation of a non-smooth transmitted signal eventually leads to discontinuities. When observed by an out-of-band receiver, the transmissions from these discontinuities may appear as strong transients with the peak power noticeably exceeding the average power, and the received signal will have a high degree of peakedness. This impulsive nature of the interference provides an opportunity to reduce its power.

***Index Terms*— Digital communications, electromagnetic interference (EMI), impulsive noise, interchannel interference, modulation, peakedness.**

I. Introduction

Let us consider a simplified measuring setup shown in Fig. 1. In the left-hand panel of the figure, the transmitter

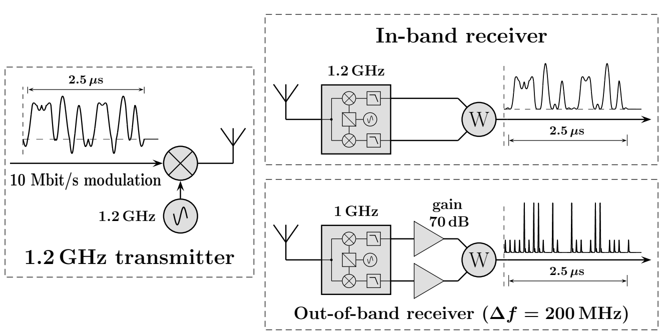

Fig. 1. Simplified setup for demonstration of the impulsive nature of interchannel interference.

emits a single 1.2 GHz tone with the amplitude modulated by a random raised cosine-shaped 10 Mbit/s message. As illustrated in the upper right-hand panel, the total instantaneous power of the in-phase and quadrature components of an in-band quadrature receiver [1] is proportional to the squared modulating signal. However, as shown in the lower right-hand panel, the total instantaneous power in an out-of-band receiver tuned to 1 GHz is an *impulsive* pulse train with a multiple of 100 ns distance between the pulses. Note that there is no apparent relationship between the magnitude of the modulating signal and the magnitude of the pulses.

Referring to a signal as *impulsive* implies that the distribution of the instantaneous power of the signal has a high degree of peakedness relative to some standard distribution, such as the Gaussian distribution. A common quantifier of peakedeness would be, for instance, the *excess kurtosis* [2]. In this paper, however, we adopt the measure of peakedness relative to a constant signal as the "excess-to-average power" ratio, and use the units "decibels relative to constant", or dBc. This measure is explained in Appendix I.

II. Impulsive Nature of Interchannel Interference

As shown in more detail in Appendix III, the signal components induced in a receiver by out-of-band communication transmitters can be impulsive. For example, if the receiver is a quadrature receiver with identical lowpass filters in the channels, the main term of the total instantaneous power of in-phase and quadrature components resulting from such out-of-band emissions may appear as a pulse train consisting of a linear combination of pulses originating at discrete times and shaped as the squared impulse response of these filters. For a single transmitter, the typical intervals between those discrete times are multiples of the symbol duration (or other discrete time intervals used in the designed modulation scheme, for example, chip and guard intervals). The non-idealities in hardware implementation of designed modulation schemes such as the non-smooth behavior of the modulator around zero, also contribute to additional discrete origins for the pulses. If the typical value of those discrete time intervals is large in comparison with the inverse bandwidth of the receiver, this pulse train will be highly impulsive.

The above paragraph can be restated using mathematical notations as follows. The total emission from various digital transmitters can be written as a linear combination of the terms of the following form:

$$x(t) = A_T(\bar{t})\, \mathrm{e}^{\mathrm{i}\omega_{\mathrm{c}} t}\,, \qquad (1)$$

where ω_{c} is the frequency of a carrier, $\bar{t} = \frac{2\pi}{T}\, t$ is *dimensionless time*, and $A_T(\bar{t})$ is the desired (or designed) complex-valued modulating signal representing a data signal with symbol duration T. Let us assume that the impulse response of the lowpass filters in both channels of a quadrature receiver is $w(t) = \frac{2\pi}{T}\, h(\bar{t})$, and that the order of the filter is larger than n so that all derivatives of $w(t)$ of order smaller or equal to $n-1$ are continuous.[1] Now let us assume that all derivatives of the same order of the modulating signal $A_T(\bar{t})$ are finite, but the derivative of order $n-1$ of $A_T(\bar{t})$ has a countable number of step discontinuities[2] at $\{\bar{t}_i\}$. Then, if $\Delta\omega = 2\pi\Delta f$ is the difference between the carrier and the receiver frequencies, and the bandwidth of the lowpass filter $w(t)$ in the receiver is much smaller than Δf, the total power in the quadrature receiver due to $x(t)$ can be expressed as[3]

$$P_x(t, \Delta f) = \frac{1}{(T\,\Delta f)^{2n}} \sum_i \alpha_i\, h\,(\bar{t} - \bar{t}_i) \sum_j \alpha_j^*\, h\,(\bar{t} - \bar{t}_j) \quad \text{for} \quad T\Delta f \gg 1\,, \tag{2}$$

where α_i is the value of the ith discontinuity of the order $n-1$ derivative of $A_T(\bar{t})$,

$$\alpha_i = \lim_{\varepsilon \to 0} \left[A_T^{(n-1)}(\bar{t}_i + \varepsilon) - A_T^{(n-1)}(\bar{t}_i - \varepsilon) \right] \neq 0\,. \tag{3}$$

A typical value of $t_{i+1} - t_i$ would be of the same order of magnitude as T. If the reciprocal of this value is small in comparison with the bandwidth of the receiver, the contribution of the terms $\alpha_i \alpha_j^* h\,(\bar{t} - \bar{t}_i)\, h\,(\bar{t} - \bar{t}_j)$ for $i \neq j$ is negligible, and (2) describes an impulsive pulse train consisting of a linear combination of pulses shaped as $w^2(t)$ and originating at $\{t_i\}$, namely

$$P_x(t, \Delta f) = \frac{1}{(T\,\Delta f)^{2n}} \sum_i |\alpha_i|^2\, h^2\,(\bar{t} - \bar{t}_i) \quad \text{for sufficiently large } T \text{ and } \Delta f\,. \tag{4}$$

This pulse train is illustrated in Panel I of Fig. 2, which shows simulated instantaneous total power response of quadrature receivers tuned to 1 GHz and 3 GHz frequencies (green and black lines, respectively) to an amplitude-modulated 2 GHz carrier of unit power. The squared impulse response of the lowpass filter in the receiver channels (30 MHz 5th order Butterworth filter [3]) is shown in the upper right corner of the panel.

The modulating signal is shown in Panel II(a) of the figure, and represents a random bit sequence at 10 Mbit/s ($T = 100$ ns). In this example, a highly oversampled FIR raised cosine filter [1] with roll-off factor 0.35 and group delay $2T$ was used for pulse shaping. A rather small group delay was chosen to make the discontinuities in the derivative more visible in the figure. Panel II(b) of Fig. 2 shows the first derivative of the modulating signal. This derivative exhibits

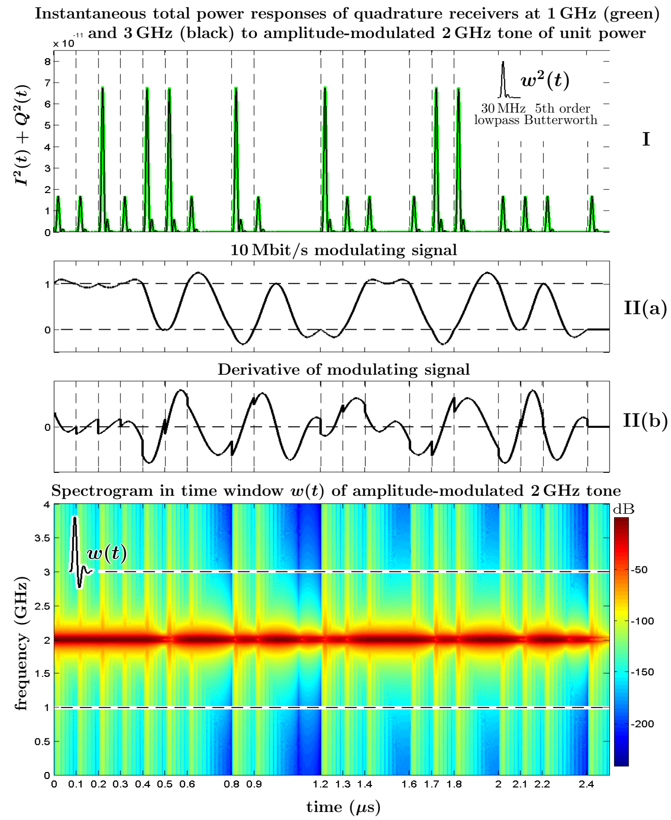

Fig. 2. Panel I of the figure shows simulated instantaneous total power response of quadrature receivers tuned to 1 GHz and 3 GHz frequencies (green and black lines, respectively) to an amplitude-modulated 2 GHz carrier of unit power. The squared impulse response of the lowpass filters in the receiver channels is shown in the upper right corner of the panel. Panels II(a) and II(b) of the figure show the modulating signal and its first derivative, respectively. For the modulating signal shown in the figure, $n = 2$ in (2). The lower panel of the figure shows instantaneous total power response of a quadrature receiver as a spectrogram in the time window $w(t)$ shown in the upper left corner of the panel.

step discontinuities at the multiple of T time intervals (at the time ticks), and thus $n = 2$ in (2).

It is important to notice that the impulsive pulse train is not necessarily caused directly by the discontinuities in the amplitude and/or phase of the transmitted signal, but rather by the discontinuities in the higher order derivatives of the modulating signal, and is generally unrelated to the magnitude of the envelope and/or the peak-to-average ratio of the transmitted signal. Thus, for instance, continuous phase modulation (CPM), while generally reducing the magnitude of the impulsive interference by increasing the order of the first discontinuous derivative by one, does not eliminate the effect altogether. This is illustrated in Appendix II.

When viewed as a function of both time and frequency, the interpretation of (2) for the total power in a quadrature receiver is a *spectrogram* [4] in the time window $w(t)$ of the term $x(t)$ of the transmitted signal. Such a spectrogram is shown in the lower panel of Fig. 2, where the horizontal dashed lines indicate the receiver frequencies 1 GHz

[1] In general, if n is the order of a causal analog filter, then $n-1$ is the order of the first discontinuous derivative of its impulse response.

[2] One will encounter discontinuities in a derivative of some order in the modulating signal sooner or later, since any physical pulse shaping is implemented using causal filters.

[3] Equation (2) will still accurately represent the total power in the quadrature receiver if the "real" (physical) modulating signal can be expressed as $A(t) = \psi(t) * A_T(t)$, where the convolution kernel $\psi(t)$ is a low-pass filter of bandwidth much larger than Δf.

and 3 GHz used in Panel I.

For a quantitative illustration of the impulsive nature of the out-of-band interference, the upper panel of Fig. 3 shows the

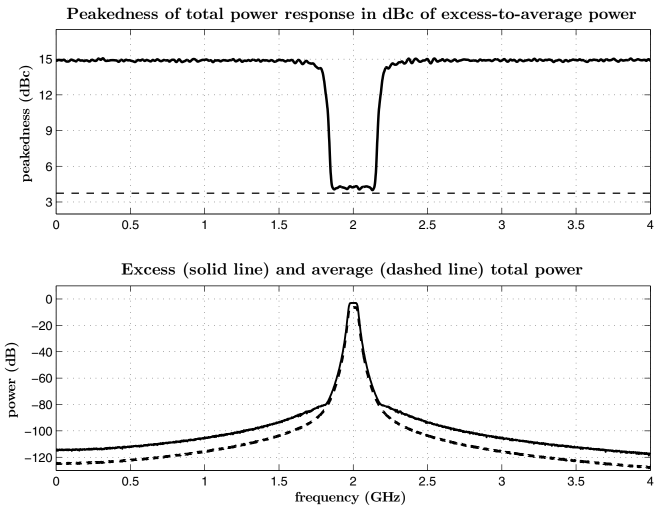

Fig. 3. Upper panel shows peakedness in dBc of the instantaneous total power response of a quadrature receiver as a function of frequency. The horizontal dashed line corresponds to the peakedness of a Gaussian distribution. The lower panel shows the total excess (solid line) and average (dashed line) power in the receiver versus frequency. The transmitted signal is a 2 GHz carrier amplitude-modulated by a random 10 Mbit/s bit stream. The impulse response $w(t)$ of the receiver and the pulse shaping of the modulating signal are as in the example shown in Fig. 2.

peakedness of the instantaneous total power in a quadrature receiver as a function of frequency for the example used in Fig. 2. The peakedness of the out-of-band signal exceeds the peakedness of the in-band signal by over an order of magnitude.

The lower panel of Fig. 3 shows, for the same examples, the total excess (solid line) and average (dashed line) power in the receiver versus frequency. The excess power of the out-of-band emissions is approximately 10 dB higher than the average power.

Given the designed properties of the transmitted signal, the out-of-band emissions can be partially mitigated by additional filtering. For example, one can apply additional high-order lowpass filtering to the modulating signal, or band-pass filtering to the modulated carrier. However, the bandwidth of those additional filters must be sufficiently large in comparison with the bandwidth of the pulse shaping filter in the modulator in order to not significantly affect the designed signal. Within that bandwidth the above analysis still generally holds, and the impulsive disturbances may significantly exceed the thermal noise level in the receiver even when the average power of the interference remains below that level.

III. Conclusion

Non-smoothness of modulation can be caused by a variety of hardware imperfections and, more fundamentally, by the very nature of any modulation scheme for digital communications. This non-smoothness sets the conditions for the interference in out-of-band receivers to appear impulsive.

If the coexistence of multiple communication devices in, say, a smartphone is designed based on the average power of interchannel interference, a high excess-to-average power ratio of impulsive disturbances may degrade performance even when operating within the specifications.

On the other hand, the impulsive nature of the interference provides an opportunity to reduce its power. Since the apparent peakedness for a given transmitter depends on the characteristics of the receiver, in particular its bandwidth, an effective approach to mitigating the out-of-band interference can be as follows: (i) allow the initial stage of the receiver to have a relatively large bandwidth so the out-of-band interference remains highly impulsive, then (ii) implement the final reduction of the bandwidth to within the specifications through nonlinear means, such as the analog filters described in [5], [6], [7], and [8]. In particular, intermittently nonlinear filters described in [9] reduce the impulsive component without detrimental effects on the transmitted message and non-impulsive noise.

Appendix I
Excess-to-Average Power Ratio as Measure of Peakedness

Consider a signal $x(t)$. Then the measure K_c of its peakedness in some time interval can be defined implicitly as the *excess-to-average power ratio*

$$\left\langle \overline{x^2}(t)\,\theta\left[\overline{x^2}(t) - K_c\right]\right\rangle = \frac{1}{2}\,, \tag{5}$$

where $\theta(x)$ is the Heaviside unit step function, $\langle\cdots\rangle$ denotes averaging over the time interval, and $\overline{x^2}(t) = x^2(t)/\langle x^2(t)\rangle$ is normalized instantaneous signal power. $K_c = 1$ for $x(t) = \text{const}$, and thus $K_{\text{dBc}} = 10\lg(K_c)$ expresses excess-to-average power ratio in units of "decibels relative to constant".

For a Gaussian distribution, K_c is the solution of

$$\Gamma\left(\frac{3}{2}, \frac{K_c}{2}\right) = \frac{\sqrt{\pi}}{4}\,, \tag{6}$$

where $\Gamma(\alpha, x)$ is the (upper) incomplete gamma function [10], and thus $K_c \approx 2.366$ ($K_{\text{dBc}} \approx 3.74$ dBc).

Appendix II
Discontinuities in Continuous Phase Modulation

For continuous phase modulation (CPM), equation (1) can be re-written as

$$x(t) = A_T(\bar{t})\, e^{i\omega_c t} = \left[A_0\, e^{i(T\,\Delta f_c)\int_{-\infty}^{\bar{t}} d\tau\, a_T(\tau)}\right] e^{i\omega_c t}\,, \tag{7}$$

where Δf_c is the frequency deviation. Then the derivative of $A_T(\bar{t})$ is

$$A'_T(\bar{t}) = i(T\,\Delta f_c) A_T(\bar{t})\, a_T(\bar{t})\,, \tag{8}$$

and, if $a_T^{(n-2)}(\bar{t})$ contains discontinuities, so does $A_T^{(n-1)}(\bar{t})$, and the rest of the analysis of this paper holds.

Appendix III
Derivation of Equation (2)

Let us examine a short-time Fourier transform of a transmitted signal $x(t)$ in a time window $w(t) = \frac{2\pi}{T}\, h(\bar{t})$ which vanishes, along with all its derivatives, outside the interval $[0,\infty[$. We will let the window function $w(t)$ represent the impulse response of an analog lowpass filter and be scaled so that $\int_0^\infty \mathrm{d}t\, w(t) = 1$.

The short-time (windowed) Fourier transform $X(t,\omega)$ of $x(t)$ can be written as

$$\begin{aligned} X(t,\omega) &= \int_{-\infty}^{\infty} \mathrm{d}\tau\, x(\tau)\, w(t-\tau)\, \mathrm{e}^{-\mathrm{i}\omega\tau} \\ &= w(t) * \left[x(t)\, \mathrm{e}^{-\mathrm{i}\omega t}\right] \\ &= w(t) * [x(t)\, \cos(\omega t)] - \mathrm{i}\, w(t) * [x(t)\, \sin(\omega t)] \\ &= I(t,\omega) + \mathrm{i}\, Q(t,\omega)\,, \end{aligned} \tag{9}$$

where the asterisk denotes convolution, and $I(t,\omega)$ and $Q(t,\omega)$ can be interpreted as the in-phase and quadrature components, respectively, of a quadrature receiver with the local oscillator frequency ω and the impulse response of lowpass filters in the channels $w(t)$.

Let us use the notation for *dimensionless time* as $\bar{t} = \frac{2\pi}{T}\, t$, and consider a transmitted signal $x(t)$ of the form

$$x(t) = A_T(\bar{t})\, \mathrm{e}^{\mathrm{i}\omega_\mathrm{c} t}\,, \tag{10}$$

where ω_c is the frequency of the carrier, and $A_T(\bar{t})$ is the desired (or designed) complex-valued modulating signal representing a data signal with symbol duration T.

The windowed Fourier transform of $x(t)$ can be written as

$$\begin{aligned} X(t,\Delta\omega) &= \int_{-\infty}^{\infty} \mathrm{d}\tau\, A_T(\bar{\tau})\, w(t-\tau)\, \mathrm{e}^{\mathrm{i}\Delta\omega\tau} \\ &= \frac{2\pi}{T} \int_{-\infty}^{\infty} \mathrm{d}\tau \left[A_T(\bar{\tau})\, h(\bar{t}-\bar{\tau})\right] \left[\frac{\mathrm{d}}{\mathrm{d}\tau} \frac{\mathrm{e}^{\mathrm{i}\Delta\omega\tau}}{\mathrm{i}\Delta\omega}\right], \end{aligned} \tag{11}$$

where $\bar{\tau} = \frac{2\pi}{T}\,\tau$ and $\Delta\omega = 2\pi\Delta f = \omega_\mathrm{c} - \omega$. Since $w(t)$ and all its derivatives vanish outside the interval $[0,\infty[$, consecutive integration by parts leads to

$$\begin{aligned} & X(t,\Delta f) = \frac{\mathrm{i}^n}{(T\,\Delta f)^n} \int_{-\infty}^{\infty} \mathrm{d}\bar{\tau}\, \mathrm{e}^{\mathrm{i}\,(T\,\Delta f)\,\bar{\tau}} \times \\ & \frac{\mathrm{d}^n}{\mathrm{d}\bar{\tau}^n} \left[A_T(\bar{\tau})\, h(\bar{t}-\bar{\tau})\right] = \frac{\mathrm{i}^n}{(T\,\Delta f)^n} \int_{-\infty}^{\infty} \mathrm{d}\bar{\tau}\, \mathrm{e}^{\mathrm{i}\,(T\,\Delta f)\,\bar{\tau}} \times \\ & \sum_{m=0}^{n} \binom{n}{m} \cdot A_T^{(n-m)}(\bar{\tau}) \cdot (-1)^m h^{(m)}(\bar{t}-\bar{\tau})\,, \end{aligned} \tag{12}$$

where $\binom{n}{m} = \frac{n!}{(n-m)!\,m!}$ is a binomial coefficient ("n choose m").

To analyze the relative contributions of the terms in (12), let us first consider the case where all derivatives of order smaller or equal to $n-1$ of the window function $w(t)$ are continuous, and all derivatives of the same order of the modulating signal $A_T(\bar{t})$ are finite, but the derivative of order $n-1$ of $A_T(\bar{t})$ has a countable number of step discontinuities at $\{\bar{t}_i\}$:

$$\alpha_i = \lim_{\varepsilon\to 0} \left[A_T^{(n-1)}(\bar{t}_i+\varepsilon) - A_T^{(n-1)}(\bar{t}_i-\varepsilon)\right] \neq 0\,. \tag{13}$$

From (13), it follows that $A_T^{(n)}(\bar{t})$ has a piecewise continuous component, as well as a singular component:

$$\begin{aligned} A_T^{(n)}(\bar{t}) &= \sum_i \alpha_i\, \delta(\bar{t}-\bar{t}_i) \\ &+ \ (\text{piecewise continuous function of } \bar{t})\,, \end{aligned} \tag{14}$$

where $\delta(x)$ is the Dirac δ-function [11].

The significance of (14) lies in the sifting (sampling) property of the Dirac δ-function:

$$\int_{-\infty}^{\infty} \mathrm{d}x\, \delta(x-x_0)\, h(x) = h(x_0) \tag{15}$$

for a continuous $h(x)$. Then substitution of (14) into (12) leads to the following expression:

$$\begin{aligned} X(t,\Delta f) = \frac{\mathrm{i}^n}{(T\,\Delta f)^n} \Bigg[& \sum_i \alpha_i\, h(\bar{t}-\bar{t}_i)\ \mathrm{e}^{\mathrm{i}\,(T\,\Delta f)\,\bar{t}_i} \\ + \int_{-\infty}^{\infty} \mathrm{d}\bar{\tau}\, \mathrm{e}^{\mathrm{i}\,(T\,\Delta f)\,\bar{\tau}} & \times (\text{continuous function of } \bar{\tau}) \Bigg]\,. \end{aligned} \tag{16}$$

The second term in the square brackets is a Fourier transform of a continuous function, and it becomes negligible in comparison with the first term as the product $T\Delta f$ increases. Thus, for the total power $P(t,\Delta f)$ in a quadrature receiver,

$$\begin{aligned} & P_x(t,\Delta f) = |X(t,\Delta f)|^2 \approx \\ & \frac{1}{(T\,\Delta f)^{2n}} \sum_i \alpha_i\, h(\bar{t}-\bar{t}_i) \sum_j \alpha_j^*\, h(\bar{t}-\bar{t}_j) \\ & \text{for} \quad T\Delta f \gg 1\,, \end{aligned} \tag{17}$$

which is equation (2) of Section II.

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