Analog approach to analysis and modeling of biometric information

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Abstract

Most of the current biometric techniques are based on logic (digital) driven approaches, which are often computationally expensive and can be found in dissonance with the continuous nature of both the biometric information itself and the human perception. Biometric information is best represented by smoothly varying quantities, with continuous range of differences within each quantity. Thus evaluation of these differences is more suitable for analysis by methods of differential calculus rather than by digital and logical means. Previous research demonstrates that modeling of the human perception should be ultimately based on continuous (analog) approaches, or, at the very least, on approaches derived from multivalued (as opposed to binary) logic.

Even though a practical outcome of biometric analysis is often of a "decision making" type, such reduction of large sets of continuous multivariate data to a single parameter characterizing the "degree of similarity" among these sets, often up to a binary ("yes" – "no") decision, can be simply done by constructing an appropriate statistic.

In this paper, we introduce a novel approach to the analysis and modeling of human image biometrics through analog representation. To illustrate the flexibility and robustness of this approach, we use an example of the so-called line objects*, representing such behavioral human characteristics as handwritten text or signatures.*

1 Introduction

The purpose of this paper is to discuss the applicability of analog and combined analog-digital techniques to model *image biometrics* of an individual. Common image biometrics include (i) physical characteristics such as fingerprints and (ii) behavioral characteristics such as handwritten text, sketches, and signatures. This paper advocates integration of analog and digital approaches to processing and modeling image biometrics through analog representation, emphasizing the fact that the measured characteristics have continuous nature. Known modeling systems discard analog information or digitize it in a form suitable for computer storage. This explains many obvious limitations of current systems such as the lack of a unified approach for image transformation operations (partially due to the intrinsic anisotropy of a discrete grid), strong dependence on the resolution of image acquisition systems, and the inability of authentication software to make use of the originally continuous nature of the signals.

In particular, the paper intends to initiate the development of software and hardware modeling tools utilizing the concept of integrated analog and digital techniques. These modeling tools would allow us to take into account the parameters of image producing and acquiring instruments (see Fig. 1), and can be deployed for image recognition, authentication, and identification in security applications. For specificity, the rest of the paper deals with such particular behavioral objects as handwritten text and signatures. However, the methods and techniques presented below can be used to model other image biometrics (fingerprints, facial characteristics, etc.).

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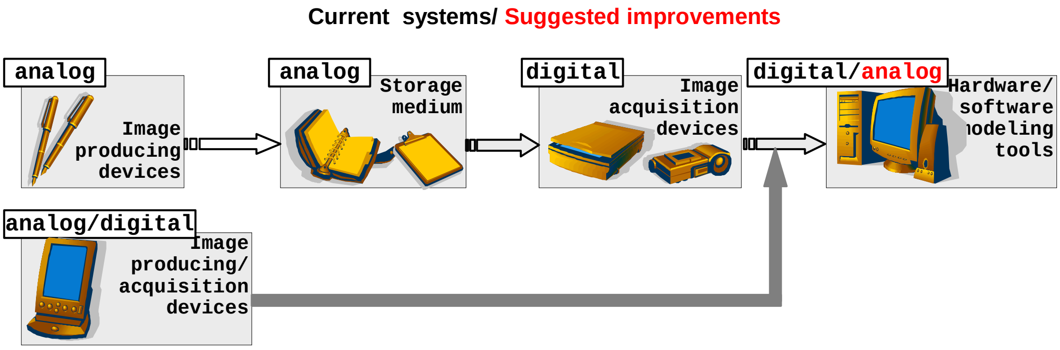

Figure 1. The sketch of an image biometric system and the place of our work

Many types of images in image biometrics can be classified as *line objects* stored on paper or in electronic medium. Line objects carry *geometric* as well as *kinematic*, *dynamic*, and other information. For example, the line shape or contour as well as its thickness represent geometric information, while characteristics such as speed of writing or exerted pressure along the drawn line represent kinematic and dynamic information, respectively. The line's color and other parameters along the contour provide additional characterization of a line object. Note that all these characteristics are continuously varying (analog) quantities, while their digital representation is given by discrete sets of data.

During the past decade, a new generation of devices, the so-called *ad hoc* scanners, has been developed to combine image producing and image acquiring stages, and to allow recording of kinematic and dynamic information [see 3, for example], which is as important for image characterization as the geometric shape. Even though the outputs of these devices are typically digital records, a continuous representation of a curve can be (re-)created by appropriate software tools. Also, since our approach has its basis in analog methodology, the algorithms for the analysis can be implemented in analog hardware. This is especially appealing for security applications, since direct analog hardware implementation eliminates most of the data transmission paths and thus reduces the possibility of tampering with the data.

The key feature of the proposed approach is the representation of a line object in terms of a *modulated linear density* $\Phi = \Phi(\boldsymbol{\eta})$, where $\boldsymbol{\eta} = (\eta_1, \ldots, \eta_n)$ is some parameter along the line object, and Φ is a non-negative (unipolar) function satisfying the following normalization condition:

$$\int_G \mathrm{d}\boldsymbol{\eta}\, \Phi(\boldsymbol{\eta}) = 1\,, \tag{1}$$

where $\mathrm{d}\boldsymbol{\eta}$ is the volume element, and the integration goes over the region G containing all values of $\boldsymbol{\eta}$. This density is an n-dimensional *continuous scalar field*, and thus can be treated as such by well established techniques of differential calculus. These techniques include integration/differentiation (including partial differentiation), various changes in coordinates (resizing, rotation, nonlinear coordinate transformations), etc. In addition, by defining the modulated linear density as a unipolar normalized quantity, we make its mathematical properties correspond to those of (probability) density functions, and thus enable the usage of various "statistical" characteristics for description of the line objects. In addition, the modulation of the line density can be viewed as a (fictitious) linear mass density, and therefore one can employ mechanical analogies (such as gyroradius and moments of inertia) for description and comparison of the line objects. In this paper, we focus on description of line objects through two-dimensional modulated densities, while the detailed description of the former in terms of one-dimensional densities is presented elsewhere [7].

The rest of the paper is organized as follows. In Section 2, we describe a simplified model of a line object, provide examples of such object's creation, and introduce the necessary definitions and terminology. Section 3 introduces the modulated linear density and describes its basic properties. Some elementary operations on line objects of image biometrics are discussed in Section 4. Section 5 provides an example of comparison of different line objects. Section 6 concludes the paper and outlines the directions of our future work.

2 Dynamic model of a line object

Let us adopt the following simplified scenario of creating a line object in an act of writing (for example, signing a document). The tip of a writing utensil follows the trajectory described by the radius (position) vector $\mathbf{r}(t)$, where t is physical time or any other ordering parameter. Changes in the exerted pressure and stroke dynamics will generally result in a different "texture," or *composition*, of the line along this trajectory. For example, the line can have varying thickness, width, and color intensity. This composition can be described by the *modulating parameter* $\mu(t)$, which we consider, without loss of generality, to be a unipolar scalar.[1] In order to enable mechanical analogies, it is sometimes convenient to interpret such modulation as the linear (pseudo-) mass density of the trajectory.

We will assume that the components of the position vector are continuous functions of time, and thus the *speed* $v(t) = |\dot{\mathbf{r}}(t)|$ is always finite. (The dot over $\mathbf{r}$ denotes time derivative.) Obviously, this notion simply reflects the physical reality of human handwriting.

If the tip of the writing utensil is infinitesimally small, it will sweep out no area, and thus the result of writing is an *ideal* line object. In reality, the tip will always have a finite size, and thus a "real-life" line object is a band of a finite width rather than an infinitesimally narrow line. With a simplification that the size and shape of the tip do not significantly change during the process, however, such a "band" object can be still described as a line, since it will be fully characterized by the trajectory of a point (e.g., the center) of the utensil's tip, and by some external modulation along the trajectory. For example, if the tip is not radially symmetric and its orientation changes during writing, the resulting change in the line's texture can be described as a simple scalar modulation. In this case, the modulation $\mu(t)$ will be the angle of rotation of the tip. However, to maintain clarity of our presentation, we will assume that the tip profile is radially symmetric and can be described by a radial function $f_d(r) > 0$,

$$2\pi \int_0^\infty \mathrm{d}r\, r f_d(r) = 1\,, \tag{2}$$

where the subscript "d" denotes the characteristic diameter of the tip.

Note that even though this description implies a dynamic model, a static image can be described in a similar manner. Furthermore, as we discuss later in more detail, the discrete data can be handled in finite differences while preserving the essentially analog philosophy of our approach. We will now proceed with the mathematical description of the modulated linear density.

[1] Vector modulation can be dealt with on a component-by-component basis.

3 Two-dimensional modulated linear density

3.1 Ideal counting (threshold crossing) density

Let us first develop a formula for an *ideal* density of a line object on a plane. Consider the task of counting the number of crossings of a point (threshold) $\mathbf{R}$ by a line described by $\mathbf{r}(t)$, during the time interval $[0, T]$. This number N can be formally expressed as

$$N = \sum_i \int_0^T \mathrm{d}t\, \delta(t - t_i)\,, \tag{3}$$

where $\delta(t)$ is the Dirac δ-function, and the summation goes over all i such that $\mathbf{r}(t_i) = \mathbf{R}$. On the other hand, the same number can be calculated as an integral over an infinitesimally small circle centered at $\mathbf{R}$, namely as

$$N = \int_0^T 2\pi\xi(t)\, |\mathrm{d}\xi(t)|\, \delta\left[\boldsymbol{\xi}(t)\right] = \int_0^T \mathrm{d}t\, |\dot{\xi}(t)|\, 2\delta\left[\xi(t)\right]\,, \tag{4}$$

where $\boldsymbol{\xi}(t) = \mathbf{R} - \mathbf{r}(t)$, and we have used the relation $\delta(\boldsymbol{\xi}) = \delta(\xi)/(\pi\xi)$ [see 2, for example]. Thus Eq. (3) can be re-written as

$$N = \int_0^T \mathrm{d}t\, |\dot{\mathbf{r}}(t)|\, 2\delta\left(|\mathbf{R} - \mathbf{r}(t)|\right)\,, \tag{5}$$

where we have used the fact that $\dot{\varepsilon}(t) = |\dot{\boldsymbol{\varepsilon}}(t)| = |\dot{\mathbf{r}}(t)|$. Integration of Eq. (5) over all possible thresholds $\mathbf{R}$ leads to

$$L = \int_0^T \mathrm{d}t\, |\dot{\mathbf{r}}(t)|\,, \tag{6}$$

which is just the total length of the trajectory. Then the ratio

$$\Phi(\mathbf{R}) = \frac{1}{L} \int_0^T \mathrm{d}t\, |\dot{\mathbf{r}}(t)|\, 2\delta\left(|\mathbf{R} - \mathbf{r}(t)|\right) \tag{7}$$

expresses the fraction of the curve's length at the point $\mathbf{R}$ to the total length of the curve,[2] and thus represents the *uniform linear density* of the curve. Notice that Eq. (7) describes a uniform linear density of an ideal writing utensil, the one with infinitesimally sharp tip. Next we extend this description to a realistic instrument, and address additional dynamic characteristics through introduction of the so-called *modulation*.

[2] "Length at a point" means the length within an infinitesimally small vicinity of the point.

3.2 Density of a line drawn by a realistic instrument

The *modulated linear density* function $\Phi(\mathbf{R})$ of a line drawn by a writing utensil with the tip profile f_d can be represented as (see, for example, [5; 4; 6])

$$\Phi(\mathbf{R}) = \frac{1}{M} \int_0^T \mathrm{d}t\, \mu(t)\, |\dot{\mathbf{r}}(t)|\, f_d\left(|\mathbf{R} - \mathbf{r}(t)|\right) , \tag{8}$$

where $\mu(t)$ is the *modulating parameter* along the line of uniform density, $|\dot{\mathbf{r}}(t)|$ is the speed of the movement of the tip, T is the duration of writing, and

$$M = \int_0^T \mathrm{d}t\, \mu(t)\, |\dot{\mathbf{r}}(t)| \tag{9}$$

is the total "pseudomass" of the trajectory. Obviously, when $\mu(t) = \text{const}$, Eq. (8) describes a *uniform* linear density. When $\mu(t) = 1$, M is just the total length of the trajectory (see Eq. (6)). The modulating parameter $\mu(t)$ can be the applied pressure, the "mass density" (e.g., thickness or brightness of the line), etc. It should be easy to see that the density function given by Eq. (8) is properly normalized according to Eq. (1).

Example *Imagine that a pen has a uniform circular tip of a diameter d. Then the tip's radial profile is described by the function*

$$f_d(r) = \frac{4}{\pi d^2}\, \theta(d - 2r) , \tag{10}$$

where $\theta(x)$ is the Heaviside unit step function. If the tip follows a trajectory described by the position vector $\mathbf{r}(t)$, and the ink flows with the constant rate $\lambda(t)$, then the density of the ink left on the paper during the time interval $[0, T]$ can be described by the function

$$\Phi(\mathbf{R}) = \frac{4}{\pi d^2 \Lambda} \int_0^T \mathrm{d}t\, \lambda(t)\, \theta\left(d - 2|\mathbf{R} - \mathbf{r}(t)|\right) , \tag{11}$$

where $\Lambda = \int_0^T \mathrm{d}t\, \lambda(t)$ is the total amount of the used ink, and the width parameter d has an obvious interpretation of the width of a drawn (straight) line. Notice that in this example the modulation is expressed as $\mu(t) = \lambda(t)/|\dot{\mathbf{r}}(t)|$, and thus the thickness of the line (the amount of ink per unit length) is inversely proportional to the speed of movement of the pen.

In many instances manipulations with a line object are based on the trajectory only, and thus assume a uniform linear density, $\mu(t) = \text{const}$ in Eq. (8). One of the exceptions is, for example, a dynamic recording of a signature, when the pressure exerted by the pen along the trajectory is also recorded. This non-uniformity in the pressure along the line is an important distinction of such a line object, and needs to be treated as a modulated density with non-constant $\mu(t)$.

4 Transformation and comparison of line objects

4.1 Center of mass, gyroradius, and inertia tensor

The comparison of biometric objects should normally be invariant to such transformations of coordinates as translation, rotation, and simple uniform scaling. A straightforward way to insure such invariance is to consider a line object as a (flat) rigid body with the mass distribution described by $\Phi(\mathbf{R})$, and use the coordinate system aligned with this body's principal axes, with the unit vector length equal to the gyroradius. See, for example, [8] or [1] for the discussion of rigid bodies and their moments of inertia.

Center of mass The center of mass $\mathbf{R}_\mathrm{c}$ is defined as

$$\mathbf{R}_\mathrm{c} = \int_{-\infty}^{\infty} \mathrm{d}^2\mathrm{r}\, \mathbf{r}\, \Phi(\mathbf{r}) , \tag{12}$$

where we have used the shortcut notation

$$\int_{-\infty}^{\infty} \mathrm{d}^2\mathrm{r} \ldots = \int_{-\infty}^{\infty} \mathrm{d}x \int_{-\infty}^{\infty} \mathrm{d}y \ldots . \tag{13}$$

Gyroradius The gyroradius R_g is defined as

$$R_\mathrm{g}^2 = \int_{-\infty}^{\infty} \mathrm{d}^2\mathrm{r}\, r^2\, \Phi(\mathbf{r}) . \tag{14}$$

Inertia tensor The components of the inertia tensor $\mathbf{I}$ are defined as

$$I_{xx} = \int_{-\infty}^{\infty} \mathrm{d}^2\mathrm{r}\, x^2\, \Phi(\mathbf{r}) , \tag{15}$$

$$I_{xy} = I_{yx} = -\int_{-\infty}^{\infty} \mathrm{d}^2\mathrm{r}\, xy\, \Phi(\mathbf{r}) , \tag{16}$$

and

$$I_{yy} = \int_{-\infty}^{\infty} \mathrm{d}^2\mathrm{r}\, y^2\, \Phi(\mathbf{r}). \tag{17}$$

Alignment of line objects Now the alignment of line objects (that is, of their respective densities) can be done by transforming the coordinates as follows: (1) translation by $-\mathbf{R}_\mathrm{c}$, (2) scaling (division) by R_g, and (3) rotation which diagonalizes the inertia tensor. Obviously, steps (2) and (3) can be interchanged [see 8; 1, for example].

4.2 Comparison: compromise between robustness and selectivity

The main purpose of representing a line in terms of its modulated density is to enable construction of various statistics for comparison of different objects, and to allow probabilistic interpretation of such comparison. Even though the density function $\Phi(\mathbf{R})$ by itself is highly sensitive to changes in the pen's trajectory (especially when line width d is small), the robustness of comparison can be greatly increased by employing an "insensitive" external instrument as explained below.

Assume that we measure the density function of Eq. (8) by an instrument with a smooth (linear) spatial impulse response $\mathcal{F}_\varrho(\mathbf{R})$, where the width parameter ϱ is indicative of the (spatial) resolution of the instrument. Then the measured density function $\Psi(\mathbf{R})$ can be expressed as

$$\Psi(\mathbf{R}) = \mathcal{F}_\varrho(\mathbf{R}) * \Phi(\mathbf{R})\,, \tag{18}$$

where the asterisk denotes convolution, and this measured density will be insensitive to small fluctuations $\delta\mathbf{r}(t)$ in the trajectory.

Now a statistic for comparison of two line objects with the measured densities Ψ_1 and Ψ_2 can be constructed in various ways. For example, one can use the following formula for estimating the "degree of similarity":

$$1 \geq Q = 1 - \frac{1}{2}\int_{-\infty}^{\infty} \mathrm{d}^2\mathrm{r}\ |\Psi_1(\mathbf{r}) - \Psi_2(\mathbf{r})| \geq 0\,, \tag{19}$$

with $Q = 1$ being a perfect match, and $Q = 0$ being a complete difference.

Let us now consider a numerical example of applying the material of Sections 3 and 4.

5 Illustrative numerical experiment

5.1 Computation in finite differences

In numerical computations, "analog" is synonymous to "high resolution." Thus, given a relatively short parametric record of a line $\{\mathbf{r}(t_i),\ \mu(t_i)\}$ (typically of order 10^3 points), we first need to convert this record into a high resolution image which can be numerically treated as a continuous object. This can be done through a convolution with a kernel f_d (representing the writing utensil and/or the reading instrument) such that its characteristic width is large in comparison with the cell of the spatial grid $\mathbf{R}_{ij}$. Then a finite difference equivalent of Eq. (8) can be written as

$$\Phi(\mathbf{R}_{ij}) = \frac{\sum_{k=1}^{N} \mu_k\, |\mathbf{r}_{k+1} - \mathbf{r}_{k-1}|\, f_d\,(|\mathbf{R}_{ij} - \mathbf{r}_k|)}{\sum_{k=1}^{N} \mu_k\, |\mathbf{r}_{k+1} - \mathbf{r}_{k-1}|}\,, \tag{20}$$

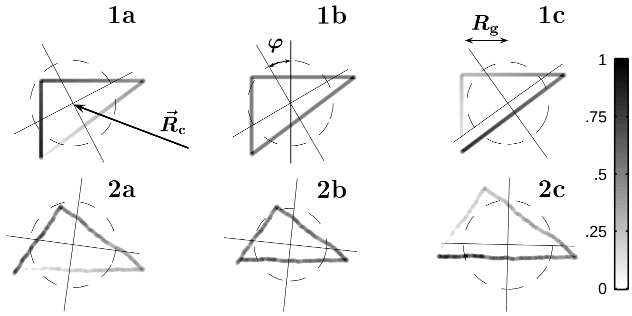

Figure 2. Original modulated linear densities of triangles with calculated principal axes and gyroradii

where $\mu_k = \mu(t_k)$ and $\mathbf{r}_k = \mathbf{r}(t_k)$. Here we assume that $t_k = (k-1)T/(N-1)$ for $1 \leq k \leq N$, and $t_0 = t_1 = 0$, $t_{N+1} = t_N = T$.

5.2 Original images and their modulated linear densities

As a simplified illustration, we have chosen images of triangular shape, as shown in Fig. 2. In Panels 1a through 1c a triangle is drawn by a point moving (in a clockwise direction) with a velocity $\mathbf{v}(t)$ ($v(t)$ = const), and in Panels 2a through 2c the velocity is rotated by some (constant) angle, multiplied by a random factor close to unity, and has an added small random component $\delta\mathbf{v}(t)$. In Panels 1a and 2a the modulation $\mu(t)$ linearly decreases, in Panels 1b and 2b it remains constant, and in Panels 1c and 2c it linearly increases. The modulated densities of the lines are computed according to Eq. (20), with the kernel f_d of the width equal to the width of the lines in Fig. 2. In the respective panels of the figure, we also show the principal axes of inertia and draw the circles (shown by the dashed lines) of gyroradii R_g, centered at the centers of mass $\mathbf{R}_\mathrm{c}$.

5.3 Transformations

Fig. 3 shows the images after the transformation consisting of the (1) additional convolution with the "reading" kernel $\mathcal{F}_\varrho$, (2) translation moving the centers of mass of the resulting densities to the origin of the coordinate system, (3) rotation aligning their principal axes of inertia with the axes of coordinates, and (4) scaling (division by R_g) normalizing their gyroradii to unity. (See Section 4.)

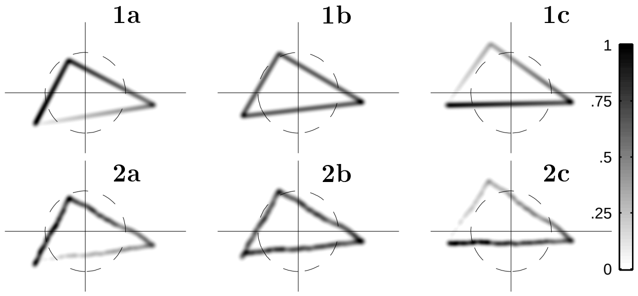

Figure 3. Modulated linear densities of triangles after translation, rotation, and scaling

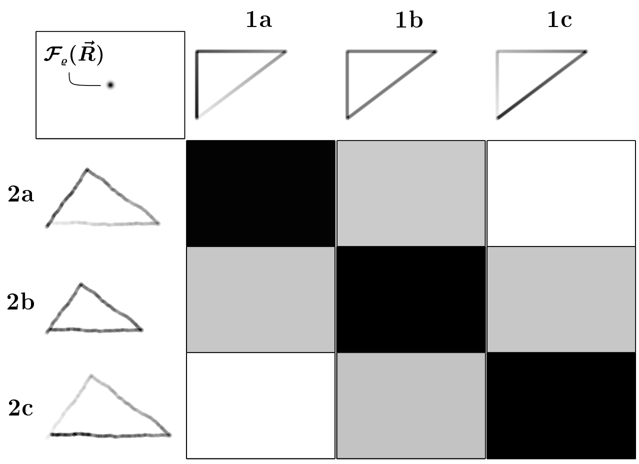

Figure 4. Comparison of densities using statistic of Eq. (19)

5.4 Comparison

Fig. 4 displays the tabulated result of comparison of the transformed densities using the statistic Q of Eq. (19). The values of Q corresponding to the specific pairs of images are indicated in grayscale at the intersections of the respective rows and columns of the table. In this example, the size ϱ of the "reading" kernel $\mathcal{F}_\varrho$ is of the same order as the width d of the writing utensil and is indicated in the upper left corner of the table. Fig. 5 illustrates the effect of the kernel's size on the robustness and selectivity of comparison.

6 Conclusion

The key component of the analog approach to the analysis of line objects presented here is the introduction of *modulated linear density*, which is a continuous function of a two-dimensional spatial coordinate. The continuity of this function allows its treatment by the operations of differential calculus and provides a means for the following fruitful reformulations of numerous analytical tasks.

Restoration of continuity Even though the basic model of the object acquisition adopted in this paper assumes a continuous parametric description of the line, a digital record can also be transformed into a continuous linear density by a convolution with a continuous kernel. Such a convolution can be performed in time as well as in the spatial domain, depending on the domain of the digitization (time and/or spatial sampling). Changing the size of the kernel is effectively equivalent to adjusting the precision of the acquisition instrument, and allows us to achieve any desired compromise between robustness and selectivity in the quantification and/or comparison algorithms.

Probabilistic interpretation Although any line object, deterministic as well as stochastic, can be transformed into a modulated linear density, the formal similarity of the latter with a *probability density function* allows us to explore probabilistic analogies and interpretations and construct a variety of "statistical" estimators of the object's properties, like those based on rank tests or linear combinations of order statistics (see a model statistic of Section 4.). This enables us to quantify similarity between a pair of line objects in a flexible way, allowing a meaningful adaptation to particular problems [see 5; 4]. For example, the quantile function

$$Q(\mathbf{x};\mathbf{a},t) = \int_{-\infty}^{\infty} \mathrm{d}^n\mathrm{r}\ \varphi(\mathbf{r};\mathbf{a},t)\ \theta\left[\varphi(\mathbf{x};\mathbf{a},t) - \varphi(\mathbf{r};\mathbf{a},t)\right] \tag{21}$$

can be given the following probabilistic interpretation: If $\mathbf{r}$ is a random variable with density function $\varphi(\mathbf{r};\mathbf{a},t)$, where $\mathbf{a}$ and t are the spatial and temporal coordinates, respectively, then, for a given $\mathbf{x}$, $Q(\mathbf{x};\mathbf{a},t)$ is the probability that $\varphi(\mathbf{x};\mathbf{a},t)$ exceeds $\varphi(\mathbf{r};\mathbf{a},t)$. This function can be a highly efficient tool in pattern recognition.

Coordinate transformation One of the main advantages of the proposed approach is that a change in a continuous density function under various nonlinear coordinate transformations can easily be calculated. This opens up, among other possible applications, the opportunity to construct such statistics for comparison of objects which are invariant to certain transformations. This is a very appealing feature in biometric analysis, since image biometric data hardly ever follow well determined geometric forms.

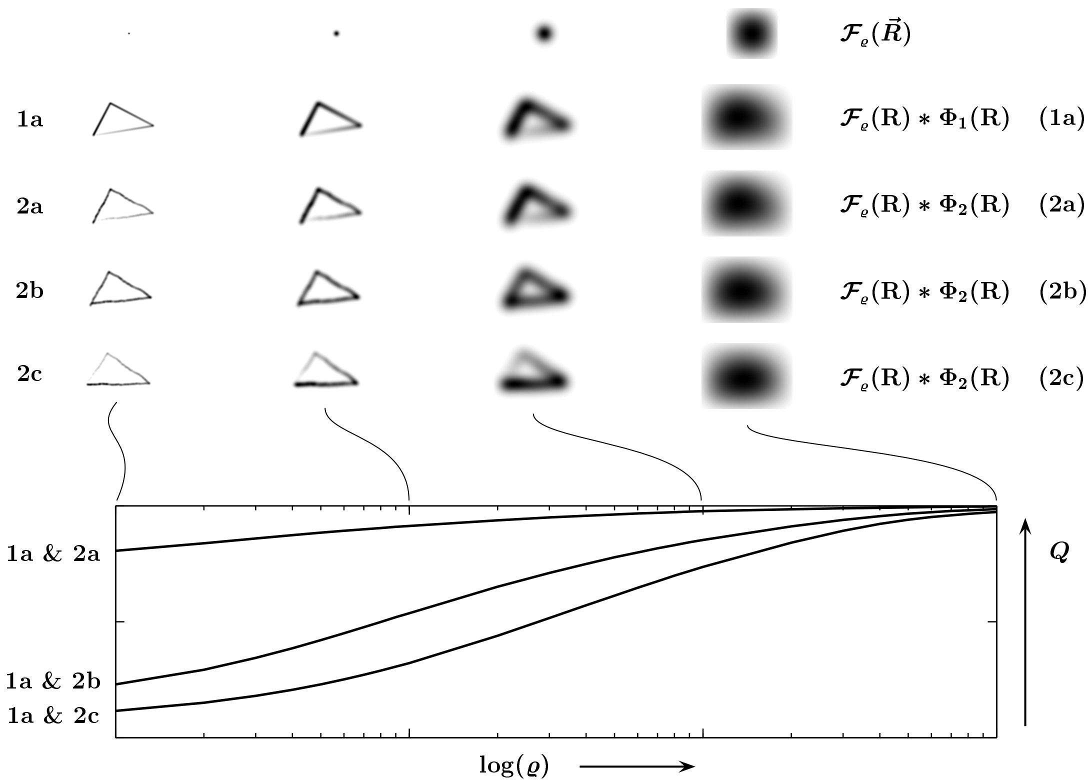

Figure 5. Compromise between robustness and selectivity

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