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Signal Analysis through Analog Representation

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Abstract

We present an approach to the analysis of signals based on analog representation of measurements. Methodologically, it relies on the consideration and full utilization of the continuous nature of a realistic, as opposed to an idealized, measuring process. Mathematically, it is based on the transformation of discrete or continuous signals into normalized continuous scalar fields with the mathematical properties of distribution functions. This approach allows a simple and efficient implementation of many traditionally digital analysis tools, including nonlinear filtering techniques based on order statistics. It also enables the introduction of a large variety of new characteristics of both one- and multi-dimensional signals, which have no digital counterparts.

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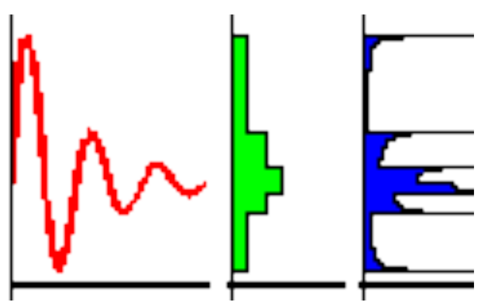
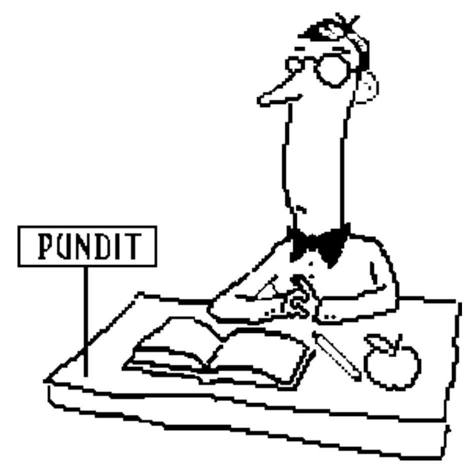
Related works in print

- A. V. Nikitin & R. L. Davidchack. Signal analysis through analogue representation. To appear in *Proc. R. Soc. Lond.* A (2002)
- A. V. Nikitin, R. L. Davidchack, & T. P. Armstrong. Analog Multivariate Counting Analyzers. To appear in *Nucl. Instr. & Meth.* A (2002)
- A. V. Nikitin & R. L. Davidchack. *Method and apparatus for analysis of variables.* To be published in 2003 under the Patent Cooperation Treaty



DISCLAIMER

Some of the definitive statements made in this presentation are intended to provoke rather than provide rigorous academic definitions. They do reflect, however, the principles to which the authors adhere in spirit if not in letter.



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Signal Analysis through Analog Representation

- Introduction
  - Why analog? What is analog?
  - Some illustrative examples & demos
  - Simplified model(s) of a measurement
    - Basic methodological principles & tools
  - Idealized and realistic threshold distributions & densities
- Extended examples
  - Nonlinear filters based on order statistics
  - Multivariate counting measurements
  - Real-time entropy-like measurements
- Summary & Discussion

Why analog? What is analog?

Why analog?

- Physical phenomena are analog, and best described by (partial) differential equations
  - Continuous measurements better relate to real physical processes
  - Continuous quantities can enter partial differential equations used in various control systems

The only obstacle to robust and efficient analog systems often lies in the lack of appropriate analog definitions and the absence of differential equations corresponding to the known digital operations. When proper definitions and differential equations are available, analog devices routinely outperform the respective digital systems, especially in nonlinear signal processing.

What is ‘analog approach to signal analysis’?

- Considering finite precision and continuity of real physical measurements in
  - Mathematical modeling of measurements
  - Treatment & analysis of data
  - Instrumentation design

and / or

- Formulating signal processing tasks in terms of continuous quantities (differential calculus)
  - E.g., in terms of continuous threshold distributions / densities

Analog solution(s) to traditionally 'digital' (discrete) problems of signal analysis offer

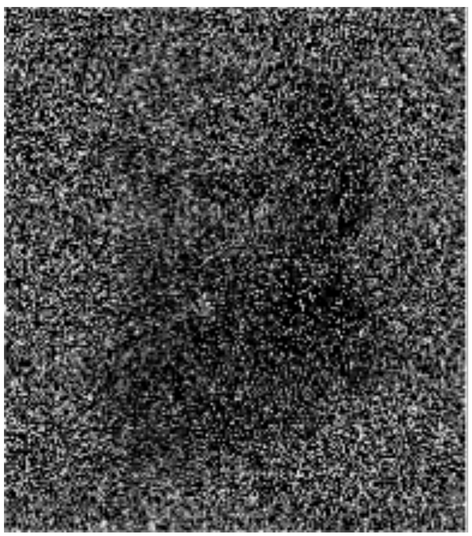
- Improved perception of the measurements through geometrical analogies
- Effective solutions of the existing computational problems of the nonlinear (such as order statistic) methods
- Extended applicability of these methods to signal analysis
- Implementation through various physical means in continuous action machines as well as through digital means or computer calculations
- Wide range of signal analysis tools which do not have digital counterparts

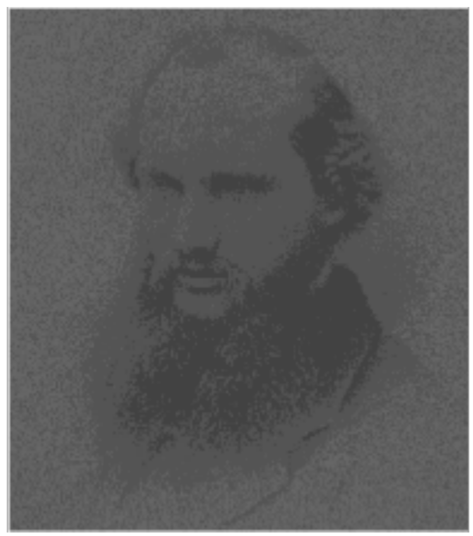
Some illustrative examples & demos

- Filtering: Noise and jitter suppression
- Visualization / Quantification / Comparison of signals
- Multivariate counting measurements

Noise suppression

Left: Noisy image. *Center:* Time averaging.
Right: Spatio-temporal analog rank filtering.





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Jitter suppression

Jitter suppression

Left: Unsteady image. *Center:* Time average.
Right: Stabilized image.

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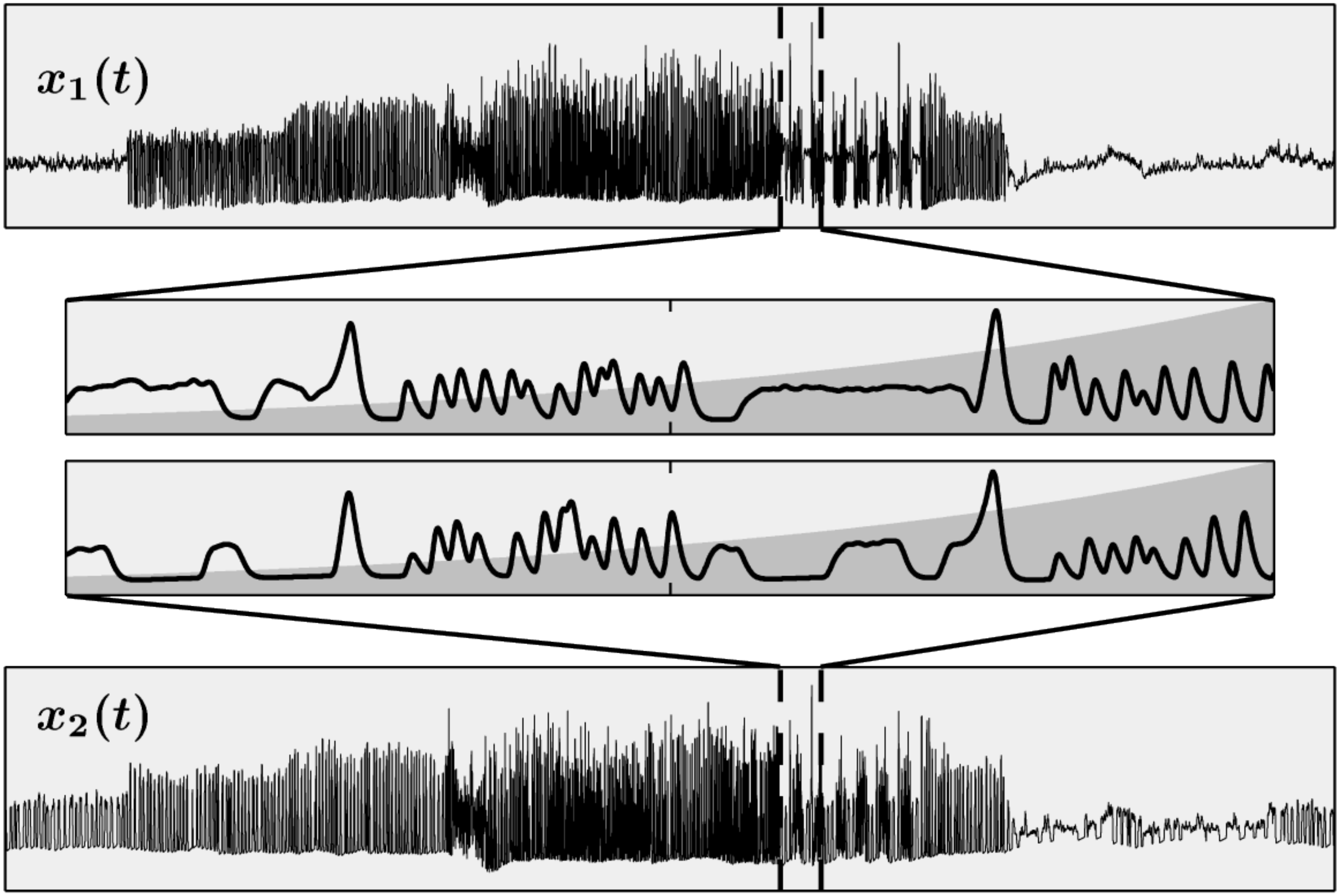
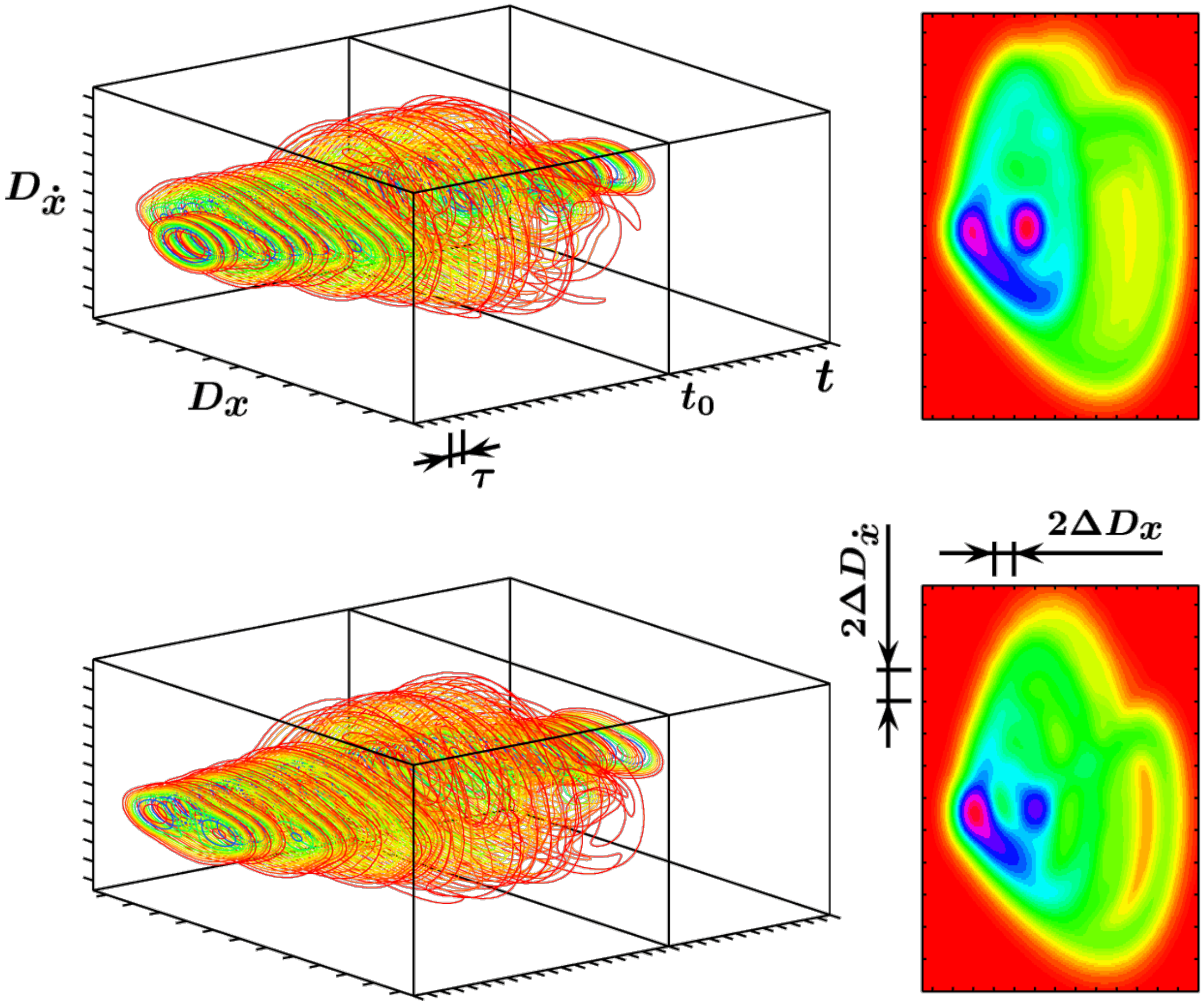
Visualization / Quantification / Comparison of signals through density flows / streams


$x_1(t)$
$x_2(t)$

Contour slices of PhS amplitude density

Slice at $t = t_0$



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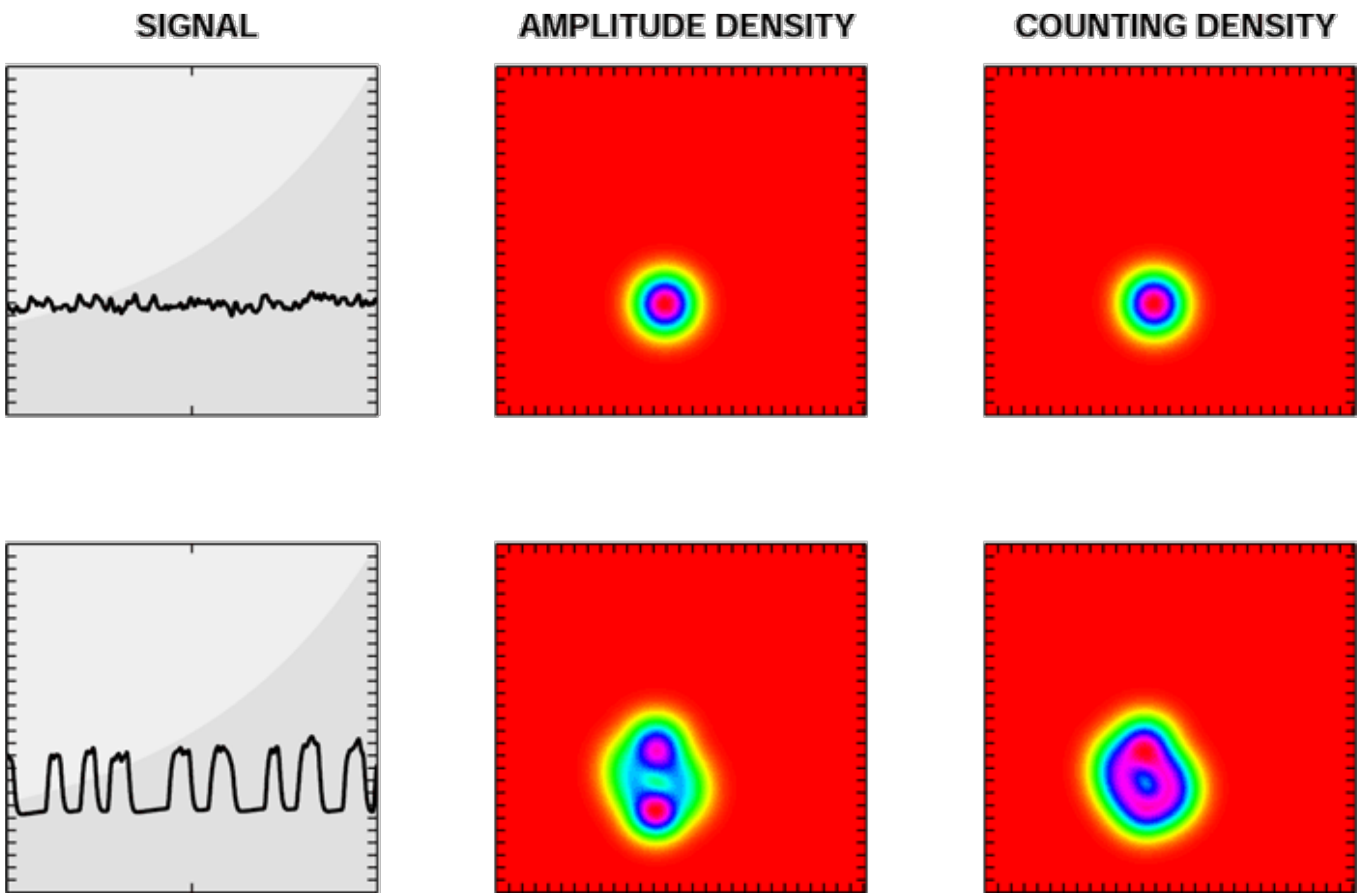
Real-time PhS density measurements



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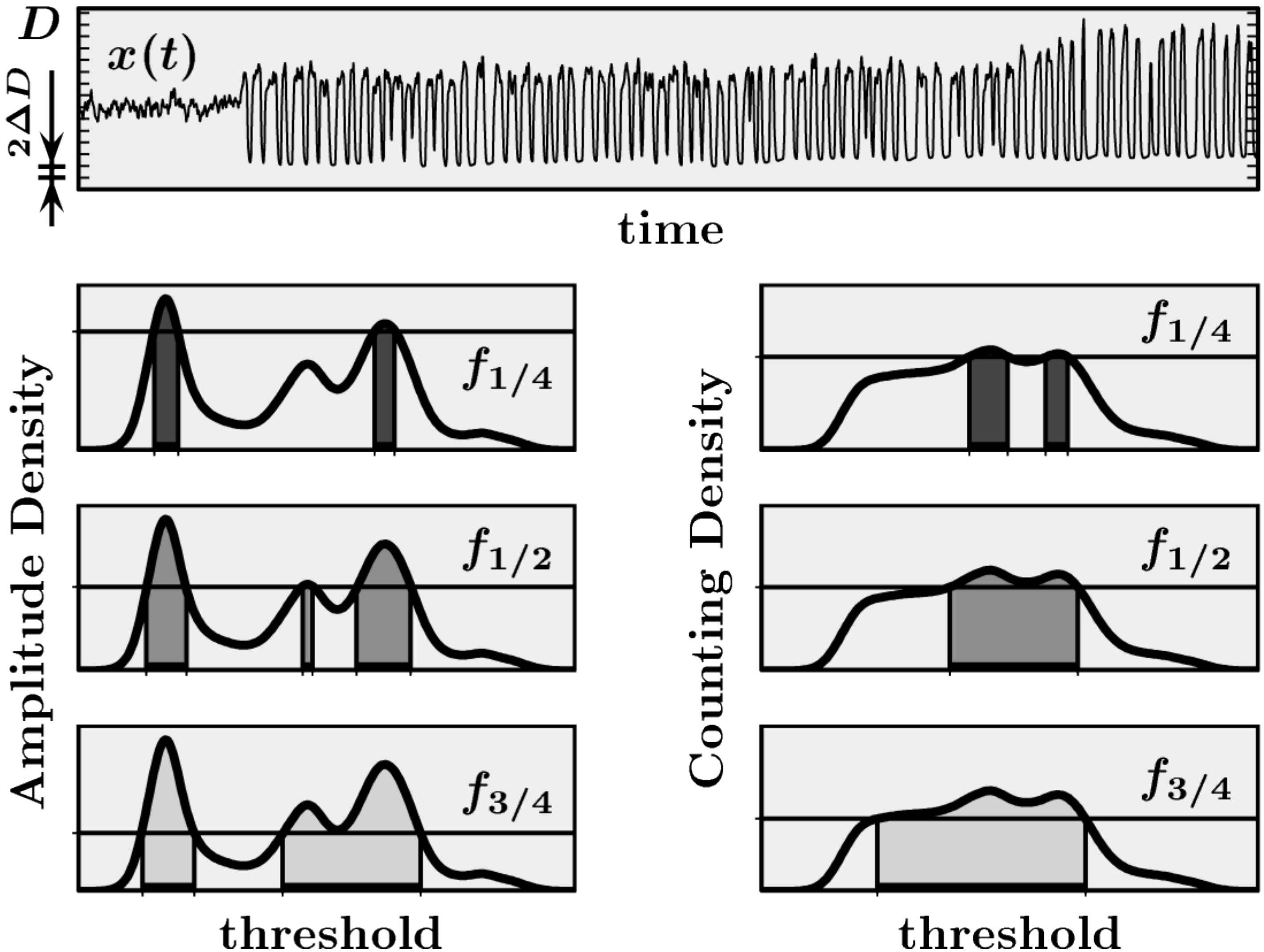
Quantile density, domain, and volume



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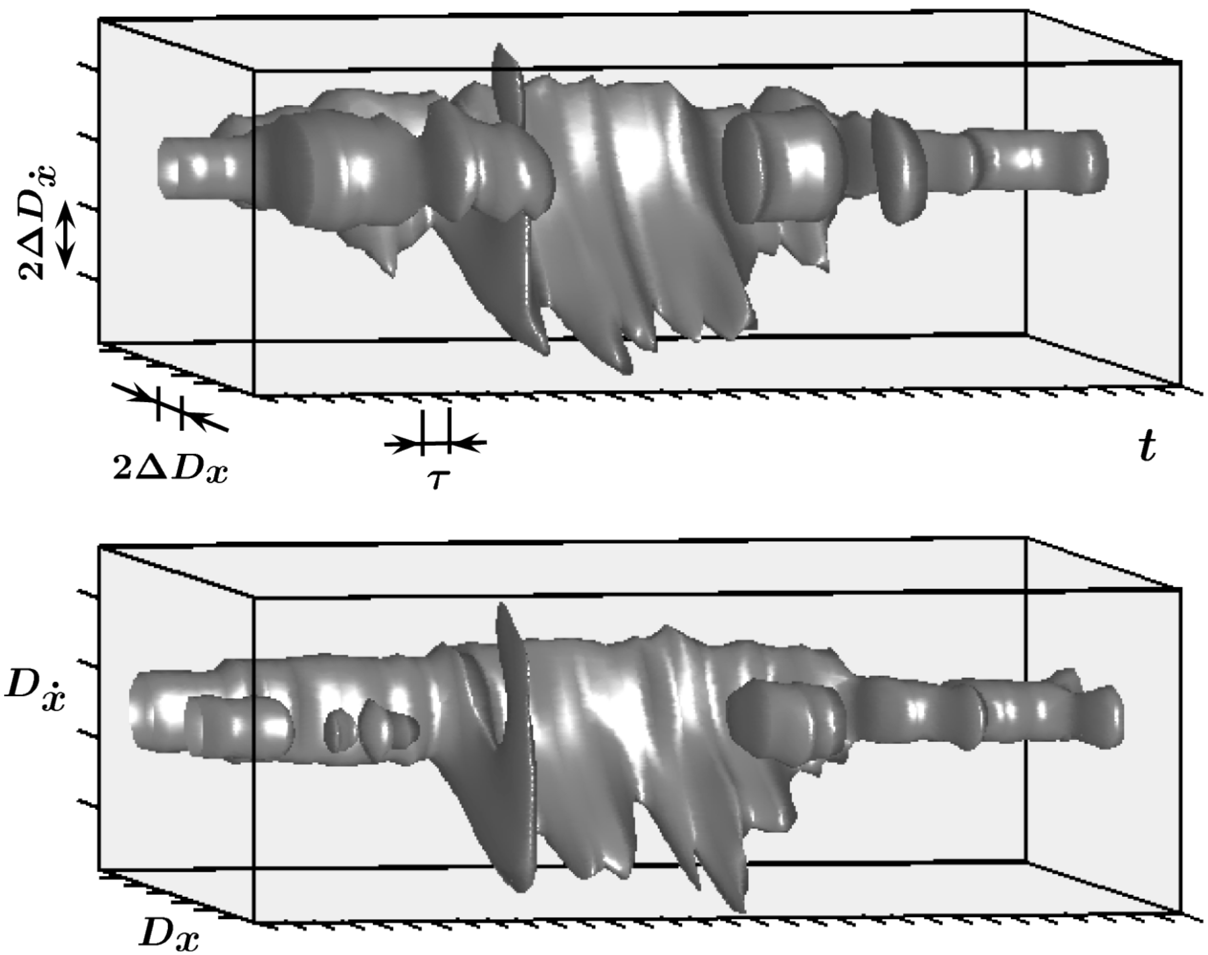
"That' s where the players are"

Quantile domains of amplitude density

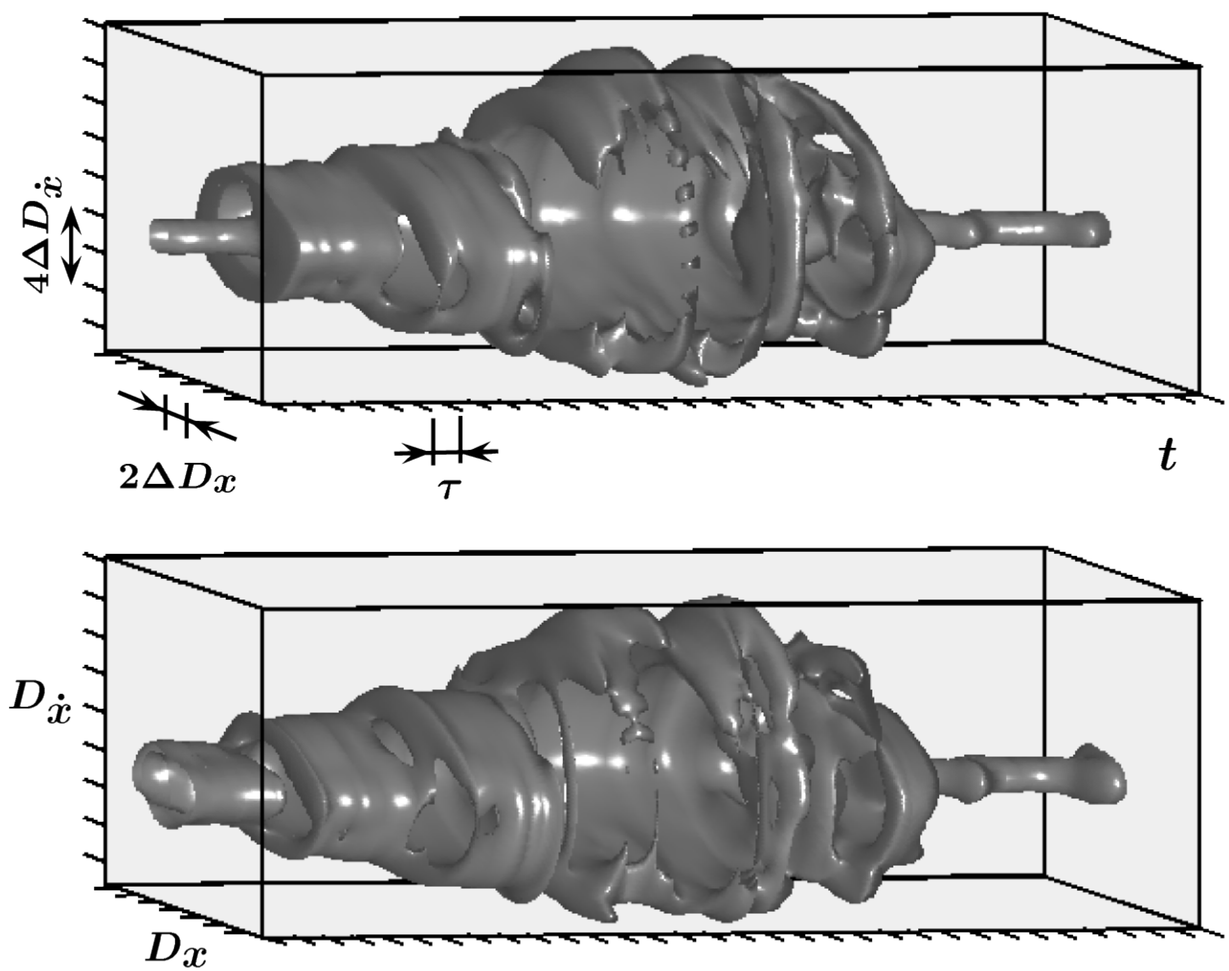
Boundary of median domain for PhS amplitude density



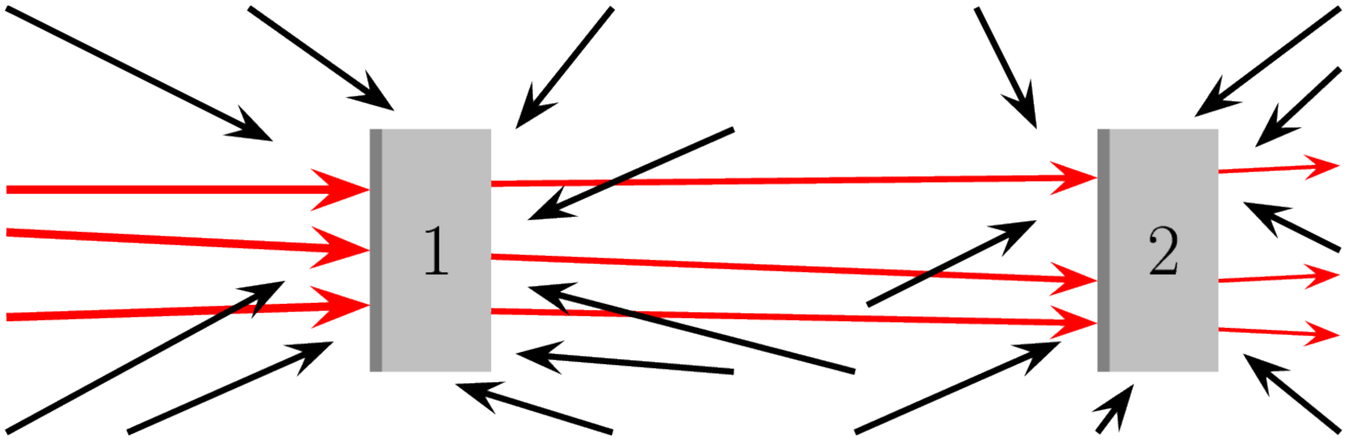
"That's where the action is"

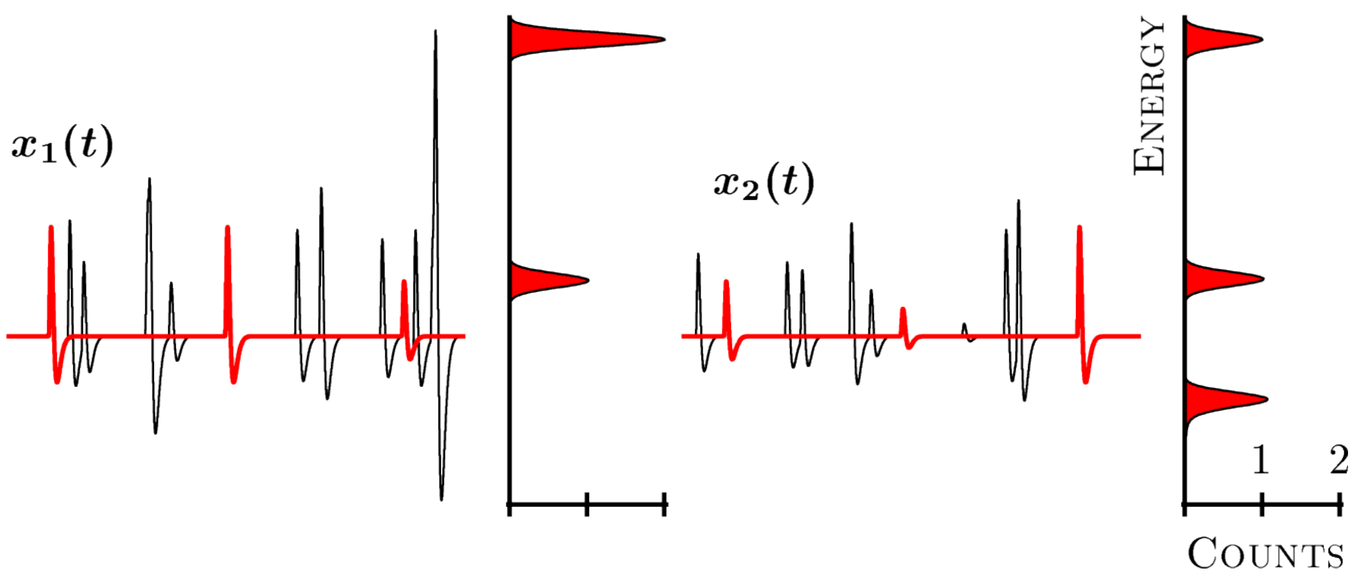
Quantile domains of counting density

Boundary of median domain for PhS counting density



Suppression of omnidirectional flux by analog coincidence counting

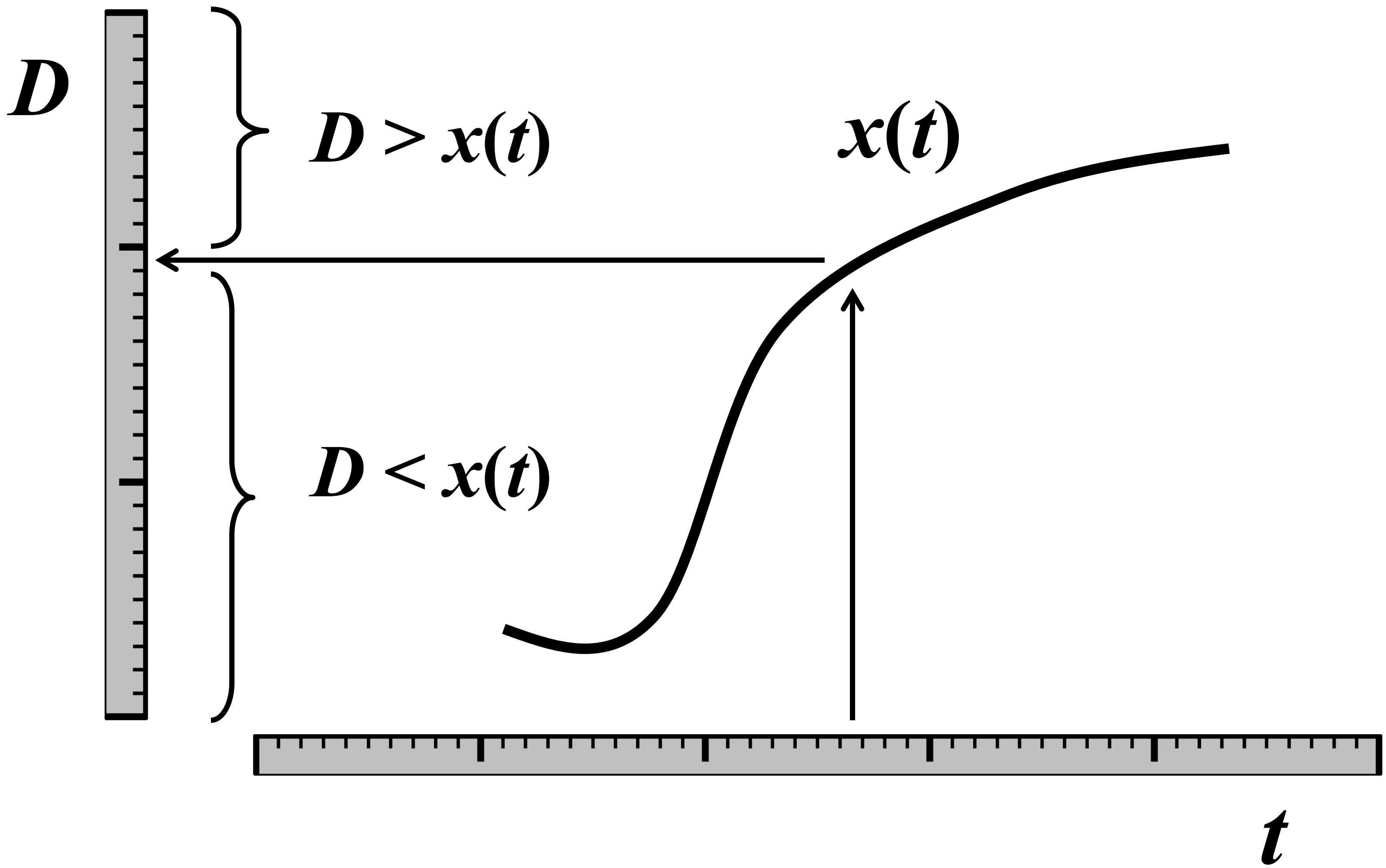




Simplified model(s) of a measurement

– Basic methodological principles & tools

Measuring ***x*(*t*)**: Value of ***x*** at time ***t***



Measurement ≡ Comparison with reference

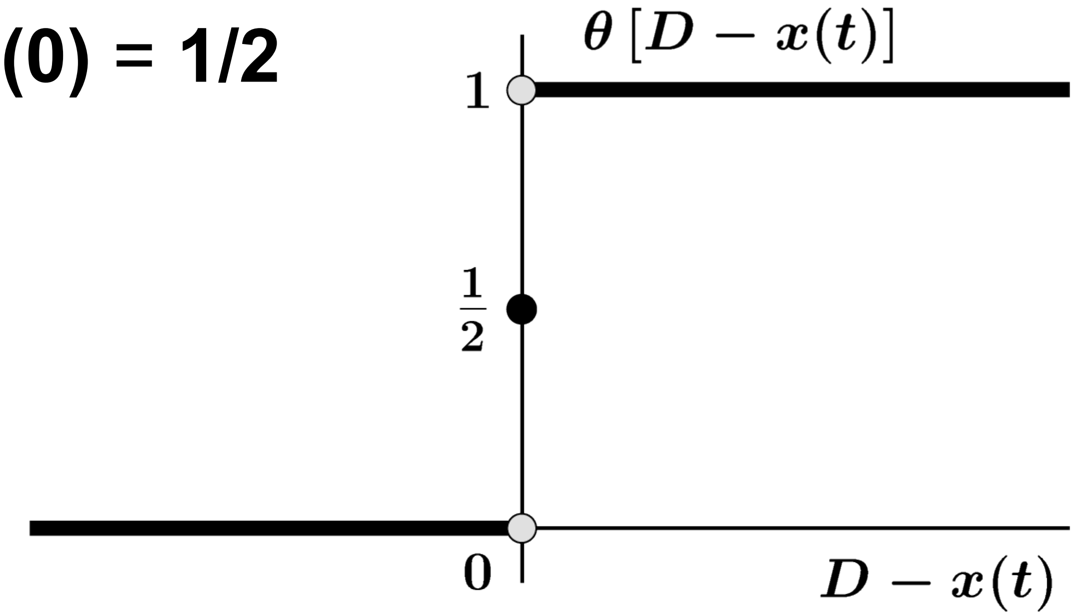
Measurement by means of an ideal discriminator:

$$\theta[D - x(t)] = \begin{cases} 1 & \text{if} \quad D > x(t) \\ 0 & \text{if} \quad D < x(t) \end{cases}$$

- θ is Heaviside unit step function
- ***D*** is displacement variable (threshold)

Output of an ideal discriminator $\theta[D-x(t)]$ indeed represents an *ideal* measurement of $x(t)$

- Can use $\theta(0) = 1/2$



- $\theta[D-x(t)] = 1/2$ describes $x(t)$ as a curve in the plane (t,D)

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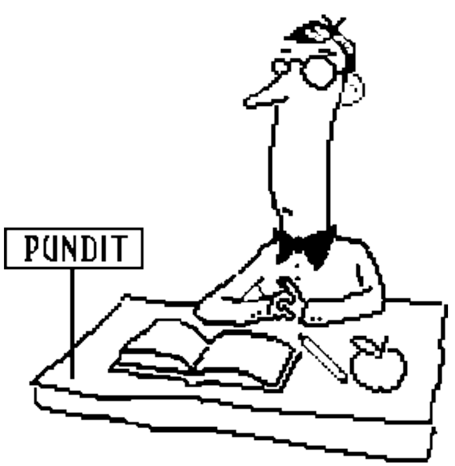
Idealization of a measurement process is convenient, and often necessary to enable meaningful mathematical treatment of the results. However, when such an idealization is carried to extremes, it becomes an obstacle in both the instrumentation design and the data analysis.



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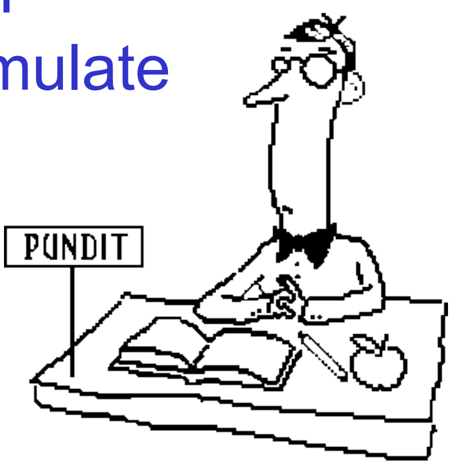
What is wrong with an ideal discriminator?
- It is “too good to be true:”

- Too fast (changes state instantaneously)
- Too accurate (capable of comparison with infinite precision)
- Too unambiguous (has no hysteresis)

Whatever device is used as a threshold discriminator, it will have finite resolution, hysteresis, time lag, and other non-ideal properties.

Accordingly, we need to replace the ideal discriminator with non-ideal functions which emulate the essential properties of the real-world measurements.



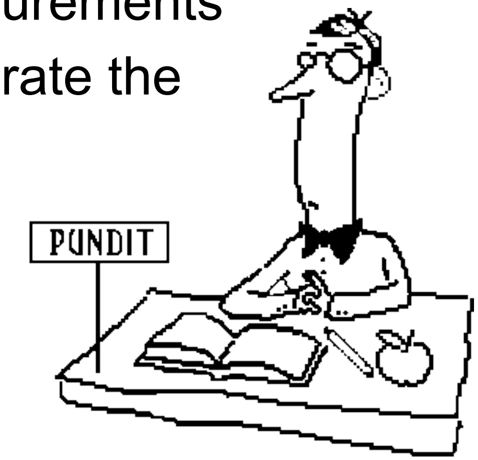
Basic methodological principles & tools

Trivial but often forgotten

1. All physical phenomena are analog in nature
2. Measurements result from interaction with instruments
3. Interpretation of data requires understanding of this interaction

=> Basic methodological principles

1. Signal processing should be formulated in terms of continuous quantities
2. Data analysis should relate to real physical measurements
3. Mathematical models & treatment should incorporate the essential properties of data acquisition systems



Before we go further

Basic tools and formulae (I): Dirac δ–function $\boldsymbol{\delta(x)}$

- Is an (even) density function satisfying the conditions

$$\delta(x) = 0 \text{ for } x \neq 0\,, \quad \int_{-\infty}^{\infty} \mathrm{d}x\, \delta(x) = 1$$

- Appears whenever one differentiates a discontinuous function, e.g.,

$$\delta(x) = \frac{\mathrm{d}}{\mathrm{d}x}\,\theta(x)$$

- While making physical sense only as part of an integrand, can be effectively used for formal algebraic manipulations, e.g.,

$$f(x)\delta(x-a) = f(a)\delta(x-a)$$

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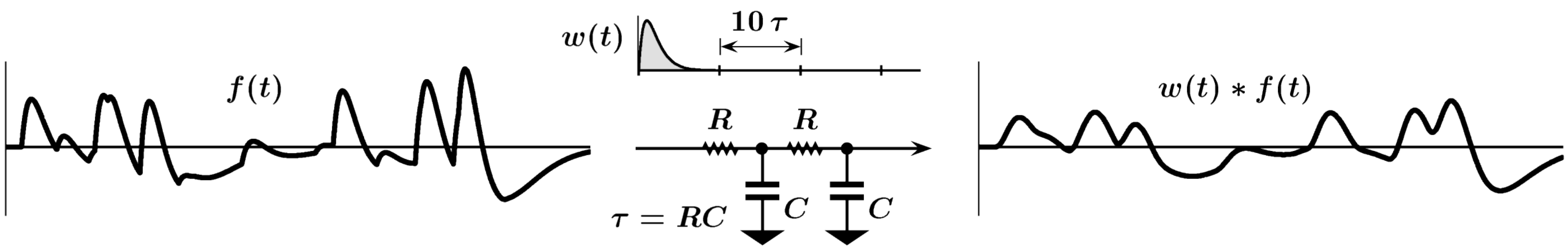
Basic tools and formulae (II):

Convolution

$$w(x) * f(x) = \int_{-\infty}^{\infty} \mathrm{d}s\, w(x-s) f(s) = \int_{-\infty}^{\infty} \mathrm{d}s\, w(s) f(x-s)$$

- If ***f*(*x*)** is an incoming signal and ***w*(*x*)** is the impulse response of an acquisition system, then ***w*(*x*)*f(*x*)** is the output signal



- Differentiation:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[w(x) * f(x)\right] = \left[\frac{\mathrm{d}}{\mathrm{d}x} w(x)\right] * f(x) = w(x) * \left[\frac{\mathrm{d}}{\mathrm{d}x} f(x)\right]$$

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Idealized and realistic threshold distributions & densities

Measurements with an 'accurate but slow' discriminator

- Can be modeled as a convolution of the output of the ideal discriminator $\boldsymbol{\theta[D\text{-}x(t)]}$ with an impulse time response function

$$w(t) \geq 0, \quad \int_{-\infty}^{\infty} \mathrm{d}t\, w(t) = 1$$

- Can be interpreted as *time dependent threshold distribution* $\boldsymbol{\Phi(D,t) = w(t) * \theta[D\text{-}x(t)]}$

Note that

- Φ(***D*,*t***) is a function of *two* variables, time ***t*** and threshold ***D***
- **0 ≤ Φ(*D*,*t*) ≤ 1** is a non-decreasing function of threshold
- For a continuous ***w*(*t*)**, **Φ(*D*,*t*)** is a continuous function of time
- The partial derivatives of **Φ(*D*,*t*)** can be written as

$$\frac{\partial}{\partial t}\Phi(D,t) = \dot{w}(t) * \theta[D - x(t)]$$

and

$$\frac{\partial}{\partial D}\Phi(D,t) = w(t) * \delta[D - x(t)] = \varphi(D,t)$$

  – ***d*** is Dirac d-function

- **φ(*D*,*t*) ≥ 0** is a *threshold density* function

Probabilistic analogies and interpretations

- Distribution $\Phi(\boldsymbol{D},\boldsymbol{t})$ and density $\varphi(\boldsymbol{D},\boldsymbol{t})$ are defined for deterministic as well as stochastic signals
- Bear formal similarity with probability function and density
- Enable the exploration of probabilistic analogies and interpretations
  - *Example:* If $\boldsymbol{s}$ is a random variable with the density $\boldsymbol{w}(\boldsymbol{t}-\boldsymbol{s})$, then $\Phi(\boldsymbol{D},\boldsymbol{t}) = \boldsymbol{w}(\boldsymbol{t}) * \theta[\boldsymbol{D}-\boldsymbol{x}(\boldsymbol{t})]$ is the probability that $\boldsymbol{x}(\boldsymbol{s})$ does not exceed $\boldsymbol{D}$
- Allow us to construct a variety of 'statistical' estimators of signal properties, e.g. those based on order statistics
  - *Example: Median* of $\boldsymbol{x}(\boldsymbol{t})$ within the window $\boldsymbol{w}$ is $\boldsymbol{D}_m = \boldsymbol{D}_m(\boldsymbol{t})$ such that $\Phi(\boldsymbol{D}_m,\boldsymbol{t}) = 1/2$ **=>** $\boldsymbol{D}_m$ is the output of the *median filter*

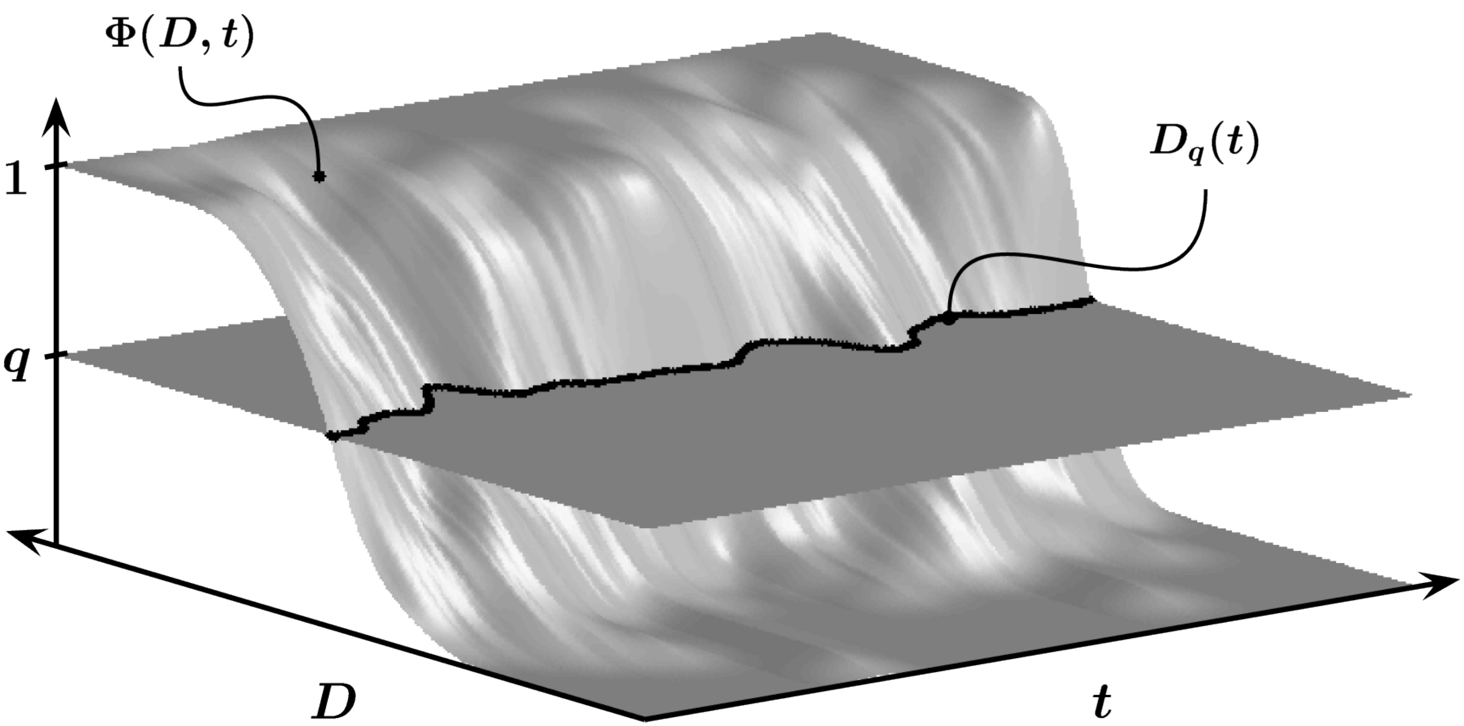
Nonlinear filters based on order statistics

Nonlinear filters based on order statistics

- There are many signal processing tasks for which digital algorithms are well known, but corresponding analog operations are hard to reproduce
- One widely recognized example is signal processing techniques based on order statistics
- Traditionally, determination of order statistics involves the operation of sorting or ordering a set of measurements
- There is no conceptual difficulty in sorting a set of discrete measurements, but it is much less obvious how to perform similar operations for continuous signals

EXAMPLE: Ideal analog quantile filters

- $D_q(t)$ is defined implicitly as $\Phi[D_q(t),t] = q$, $0< q <1$
- $\Phi(D,t)$ is a surface in the three-dimensional space (t,D,Φ)
- $D_q(t)$ is a level (or contour) curve obtained from the intersection of the surface $\Phi(D,t)$ with the plane $\Phi = q$



- From the sifting property of the Dirac δ-function:

$$D_q(t) = \int_{-\infty}^{\infty} \mathrm{d}D\, D\, \delta[D - D_q(t)] = \int_{-\infty}^{\infty} \mathrm{d}D\, D\, \varphi(D,t)\, \delta\left[\Phi(D,t) - q\right]$$

  – Leads to analog L-filters and α-trimmed mean filters

- From a differential equation of a level curve:

$$\frac{\mathrm{d}D_q}{\mathrm{d}t} = -\frac{\partial\Phi/\partial t}{\partial\Phi/\partial D_q} = -\frac{\partial\Phi(D_q,t)/\partial t}{\varphi(D_q,t)}$$

  – $\boldsymbol{D_q(t)}$ will follow the level curve given a proper initial condition
  – Enables implementation of quantile filters by analog feedback circuits

Analog L-filters and α-trimmed mean filters

- Linear combination of quantile filters:

$$D_L(t) = \int_0^1 \mathrm{d}q\, W_L(q)\, D_q(t) = \int_{-\infty}^{\infty} \mathrm{d}D\, D\, \varphi(D,t)\, W_L\left[\Phi(D,t)\right]$$

  – W_L is some (normalized) weighting function

- Particular choice of W_L as a boxcar probe b_a of width **1-2*a*** centered at ½ leads to α-trimmed mean filters:

$$\overline{D}_\alpha(t) = \int_{-\infty}^{\infty} \mathrm{d}D\, D\, \varphi(D,t)\, b_\alpha\left[\Phi(D,t)\right]\,, \quad 0 \le \alpha < 1/2$$

  – Running mean filter when ***a* = 0**
  – Median filter when ***a* = 1/2**

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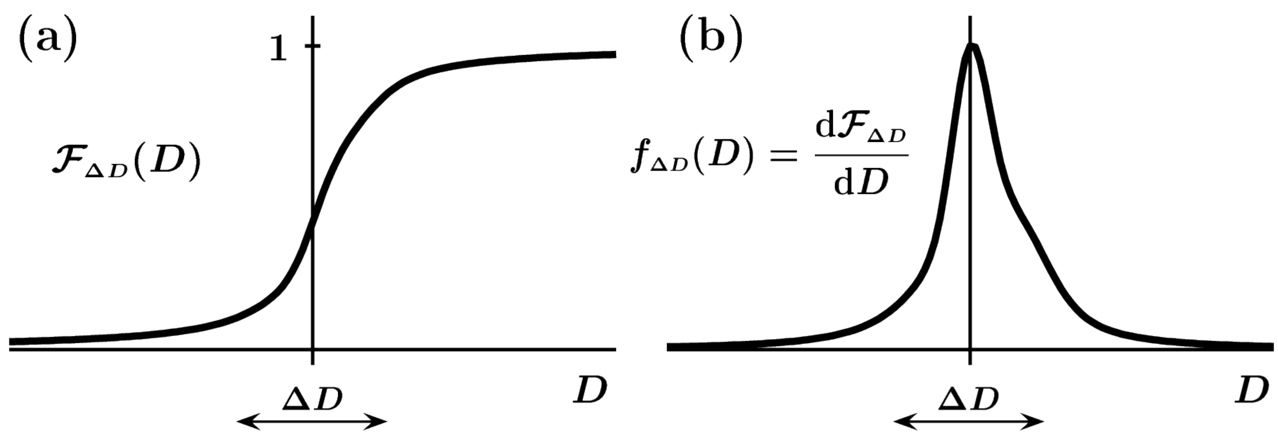
Feedback analog rank filters

Ideal filter $$\frac{\mathrm{d}D_q}{\mathrm{d}t} = -\frac{\dot{w}(t) * \theta[D_q - x(t)]}{w(t) * \delta[D_q - x(t)]}$$ is impractical:

- Denominator $\varphi(D_q, t) = w(t) * \delta[D_q - x(t)]$ cannot be directly evaluated
- Quantile order ***q*** is employed only via the initial conditions **=>**
  - Any deviation from the initial condition will result in different order filter
  - Noise will cause the output to drift away from the chosen value of ***q***
- Convolution integrals in the right-hand side need to be re-evaluated (updated) for each new value of D_q

'Real' discriminators and probes

- δ-function in the expression for the threshold density φ**(*D*,*t*)** is the result of the infinite-precision idealization of measurements
- All physical observations are limited to a finite resolving power, and the only measurable quantities are weighted means over nonzero intervals **=>**
- A more realistic discriminator is a continuous function $\mathbf{F}_{\Delta D}(D)$ which changes monotonically from **0** to **1** so that most of this change occurs over some characteristic range of threshold values ΔD



- Ideal density cannot be measured / evaluated

$$\varphi(D,t) = w(t) * \delta[D - x(t)] = \sum_i \frac{w(t_i)}{|\dot{x}(t_i)|}$$

  – sum goes over all $\boldsymbol{t_i}$ such that $\boldsymbol{x(t_i) = D}$

  => we need to know all threshold crossings within $\boldsymbol{w(t)}$

  => $\boldsymbol{\varphi(D,t)}$ is infinite at each extremum of $\boldsymbol{x(t)}$ within $\boldsymbol{w(t)}$

- 'Real' density can be viewed as the threshold average of the ideal density with respect to the test function $\boldsymbol{f_{\Delta D}(D)}$:

$$f_{\Delta D}[D - x(t)] = \int_{-\infty}^{\infty} \mathrm{d}r \, f_{\Delta D}(D - r)\, \delta[r - x(t)]$$

- No problem measuring / evaluating the real density $$\boxed{\varphi(D,t) = w(t) * f_{\Delta D}[D - x(t)]}$$

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Stability with respect to quantile values

$$\frac{\mathrm{d}D_q}{\mathrm{d}t} = -\frac{\partial\Phi(D_q,t)/\partial t}{\varphi(D_q,t)} + \nu\,[q - \Phi(D_q,t)], \qquad \nu > 0$$

- Parameter ν is the characteristic convergence speed (in units 'threshold per time')
- Since **Φ(*D,t*)** is a monotonically increasing function of ***D*** for all ***t*,** the added term will ensure convergence of the solution to the chosen quantile order ***q*** regardless of the initial condition
- Consideration of the inertial properties of an acquisition system leads to a simple 'natural' choice for ν

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Design simplification from the consideration of a realistic measurement process

- Inertial properties of many physical sensors are well represented by the transient characteristic

$$H_\tau = \theta(t)(1 - \mathrm{e}^{-t/\tau})$$

  - τ is characteristic response time

=>

- Total impulse time response of a typical measuring device is $w(t) = h_\tau(t) * w_T(t)$
  - $h_\tau(t) = \theta(t)\,\frac{1}{\tau}\,\mathrm{e}^{-t/\tau}$
  - w_T is desired (or designed) impulse response
- Time derivative of ***w*(*t*)** can be expressed as

$$\frac{\mathrm{d}w}{\mathrm{d}t} = \frac{\mathrm{d}h_\tau}{\mathrm{d}t} * w_T(t) = \frac{1}{\tau}\,[\delta(t) - h_\tau(t)] * w_T(t) = \frac{1}{\tau}\,[w_T(t) - w(t)]$$

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We can choose the characteristic speed of convergence ν as

$$\nu = \{\tau\, h_\tau(t) * w_T(t) * f_{\Delta D}\,[D_q - x(t)]\}^{-1}$$

$$\Rightarrow \quad \boxed{\frac{dD_q}{\mathrm{d}t} = \frac{q - w_T(t) * \mathcal{F}_{\Delta D}\,[D_q - x(t)]}{\tau\, h_\tau(t) * w_T(t) * f_{\Delta D}\,[D_q - x(t)]}}$$

- ***q*** is quantile order, **0< *q* <1**
- $\boldsymbol{h_\tau * w_T = w}$ is the total impulse time response
- $\mathbf{F}_{\Delta D}$ and $\boldsymbol{f_{\Delta D}}$ are the discriminator and its associated probe

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Final step: Approximations for feedback circuits

The right-hand side of the equation for $\mathrm{d}D_q/\mathrm{d}t$ can be approximated in various ways, e.g.

- $$\frac{\mathrm{d}D_q}{\mathrm{d}t} \approx \frac{q - \sum_k w_k \, \mathcal{F}_{\Delta D}\left[D_q(t) - x(t - t_k)\right]}{\tau \, h_\tau(t) * \sum_k w_k \, f_{\Delta D}\left[D_q(t) - x(t - t_k)\right]}$$
  - $$w(t) = h_\tau(t) * \sum_k w_k \, \delta(t - t_k) \,, \quad \sum_k w_k = 1$$
- Analog rank selector among N signals $x_k(t)$:

$$\frac{\mathrm{d}D_q}{\mathrm{d}t} = \frac{N q - \sum_{k=1}^{N} \mathcal{F}_{\Delta D}\left[D_q(t) - x_k(t)\right]}{\tau \, h_\tau(t) * \sum_{k=1}^{N} f_{\Delta D}\left[D_q(t) - x_k(t)\right]}$$

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(1) Introduction of real discriminators and **(2)** consideration of inertial properties of measuring devices (e.g., $\boldsymbol{w = h_\tau * w_T}$) leads to various generalized rank filters for continuous signals, including simple and efficient implementations of feedback quantile filters / selectors

$$\theta(D) \;\to\; \mathcal{F}_{\Delta D}(D), \quad \delta(D) \;\to\; f_{\Delta D}(D)$$

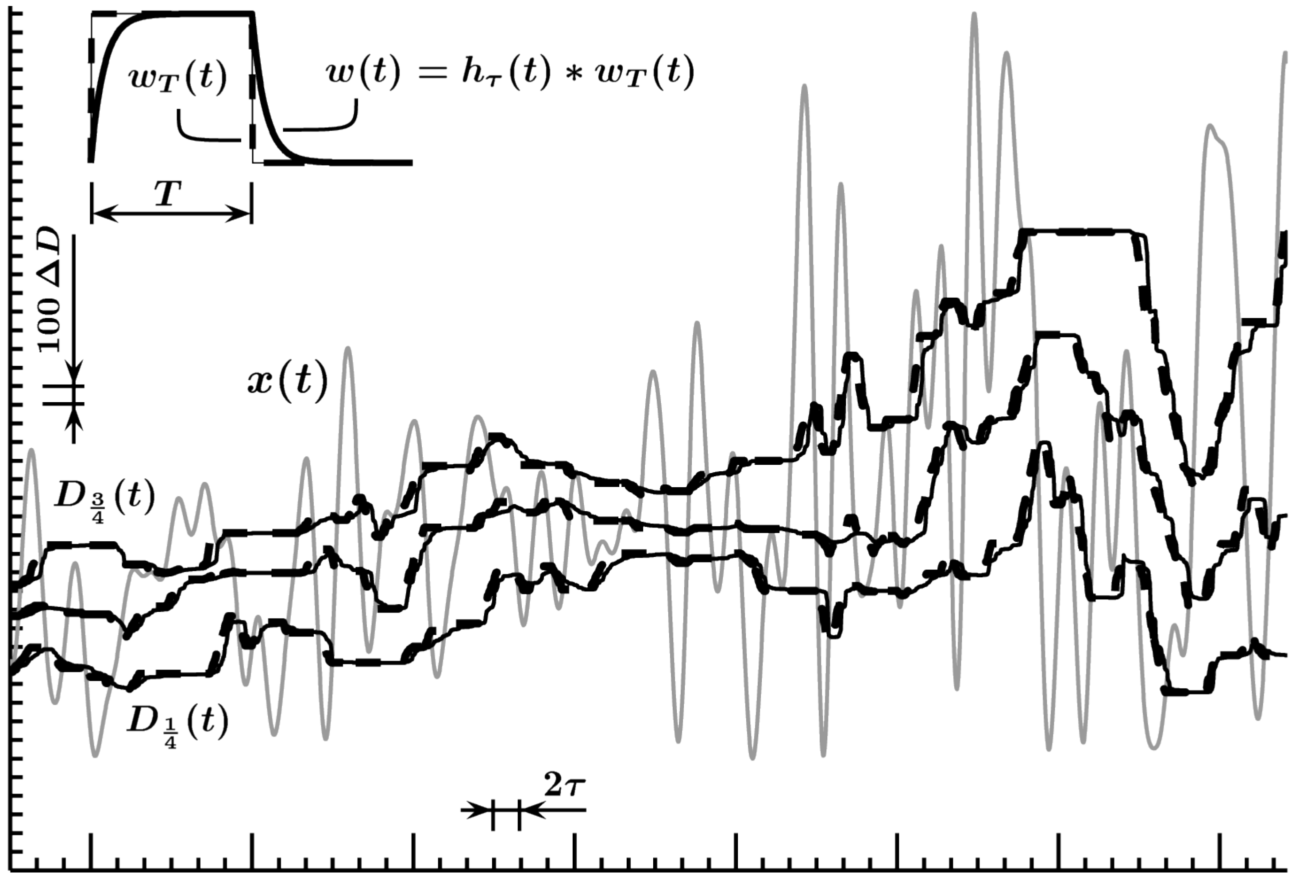
$$w(t) \;\to\; h_\tau(t) * w_T(t),$$

$$\text{where } h_\tau(t) = \theta(t)\,\frac{1}{\tau}\,\mathrm{e}^{-t/\tau}$$

Quartile outputs (solid black lines) of an analog quantile filter

$$\frac{\mathrm{d}D_q}{\mathrm{d}t} = \frac{q - \sum_k w_k \, \mathcal{F}_{\Delta D}\left[D_q(t) - x(t - t_k)\right]}{\tau \, h_\tau(t) * \sum_k w_k \, f_{\Delta D}\left[D_q(t) - x(t - t_k)\right]}$$

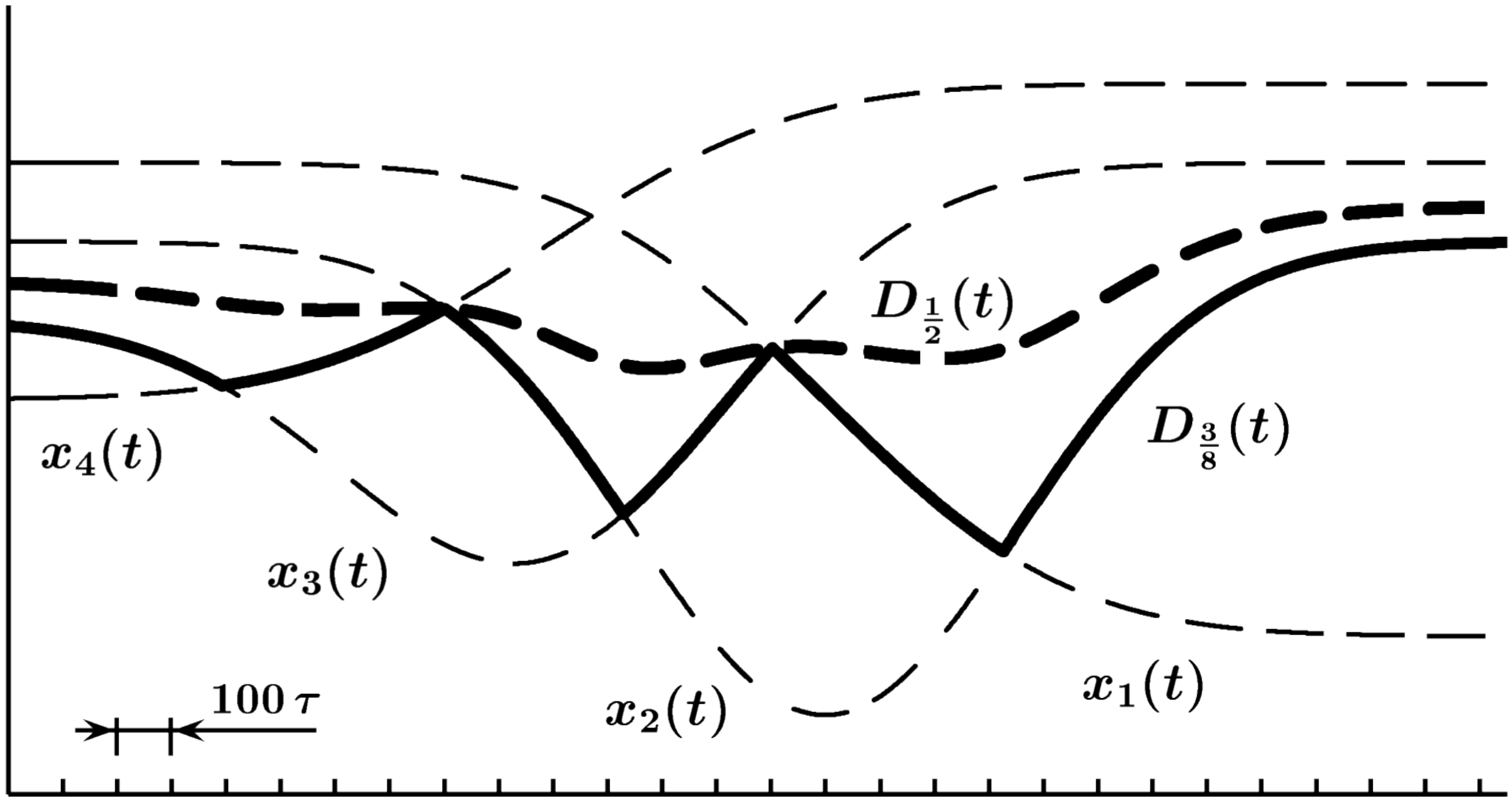
- Respective outputs of a rank filter in a rectangular window $\boldsymbol{w_T}$ are shown by dashed lines



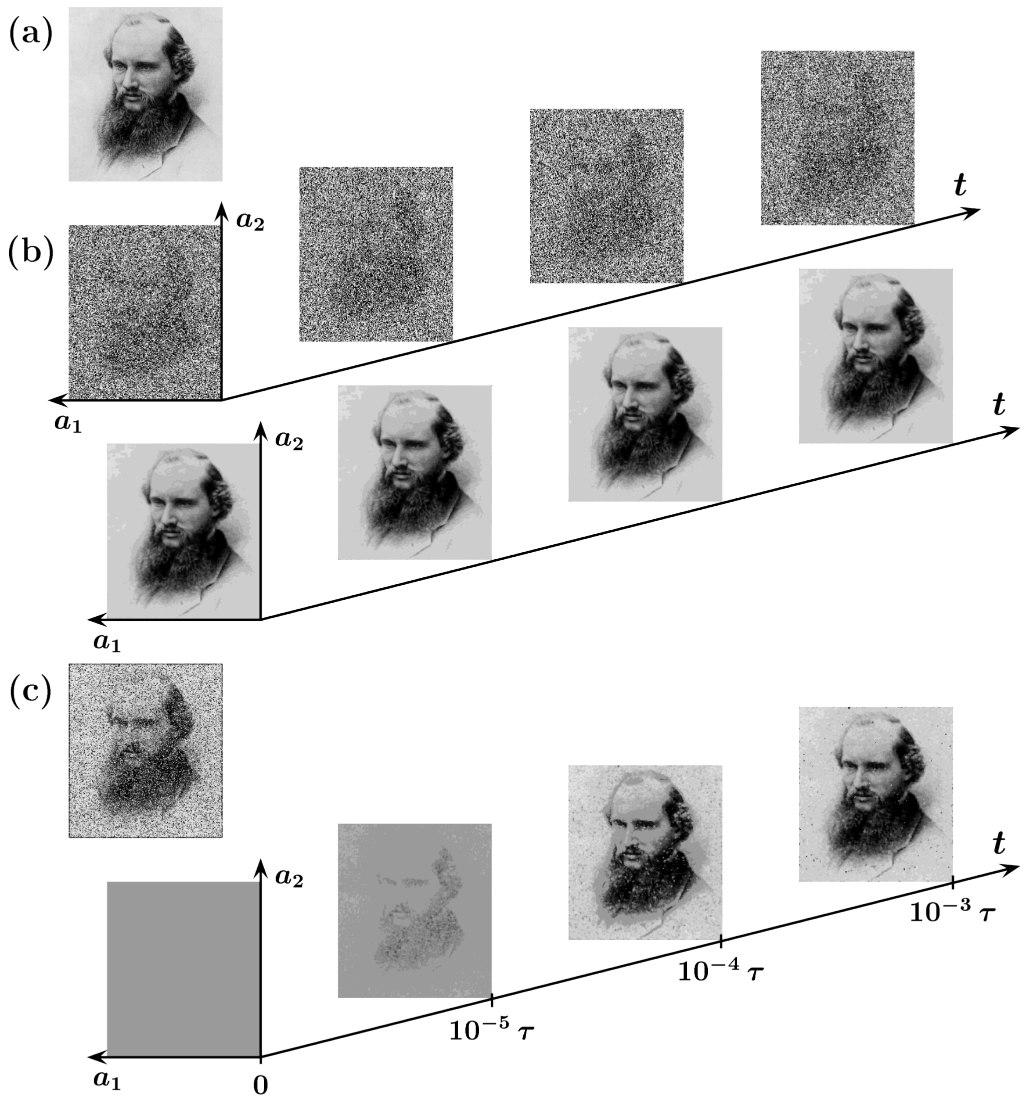
Analog rank selector $\frac{\mathrm{d}D_q}{\mathrm{d}t} = \frac{4q - \sum_{k=1}^{4} \mathcal{F}_{\Delta D}\left[D_q(t) - x_k(t)\right]}{\tau\, h_\tau(t) * \sum_{k=1}^{N} f_{\Delta D}\left[D_q(t) - x_k(t)\right]}$

for four signals (***x*$_1$(*t*)** through ***x*$_4$(*t*)**, thin dashed lines)

- Thick dashed line shows the median (***q*=1/2**)
- Solid line shows the 3rd octile (***q*=3/8**)

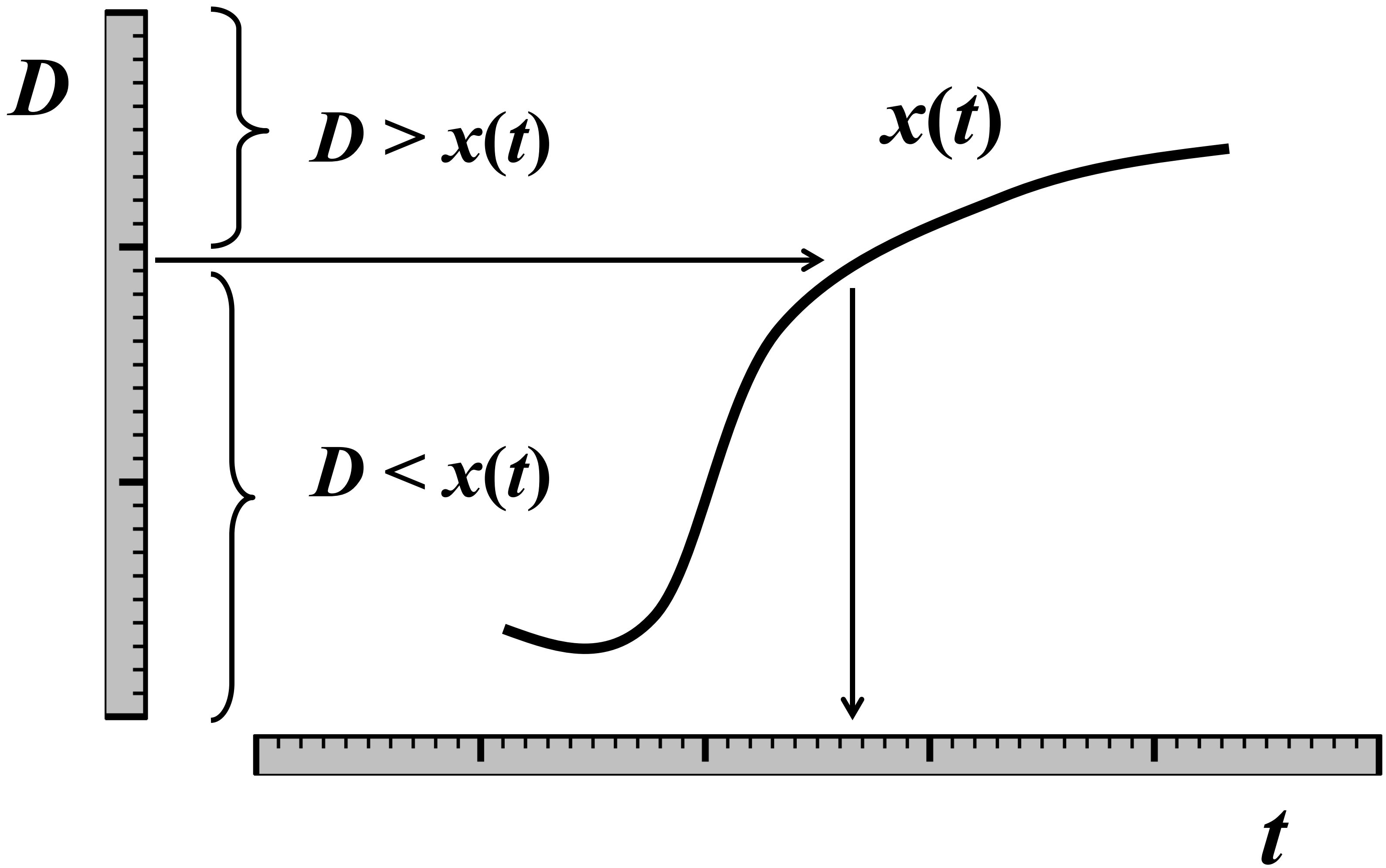


Removing static and dynamic impulse noise from a monochrome image by the filter $\dfrac{\mathrm{d}D_q(\mathrm{a},t)}{\mathrm{d}t} = \dfrac{q - f_{\mathrm{R}}(\mathrm{a}) * \mathcal{F}_{\Delta D}\left[D_q - x(\mathrm{a},t)\right]}{\tau\, h_\tau(t) * f_{\mathrm{R}}(\mathrm{a}) * f_{\Delta D}\left[D_q - x(\mathrm{a},t)\right]}$



Multivariate counting measurements

Counting measurements: Crossing of ***D*** by ***x*(*t*)**



Multivariate counting measurements

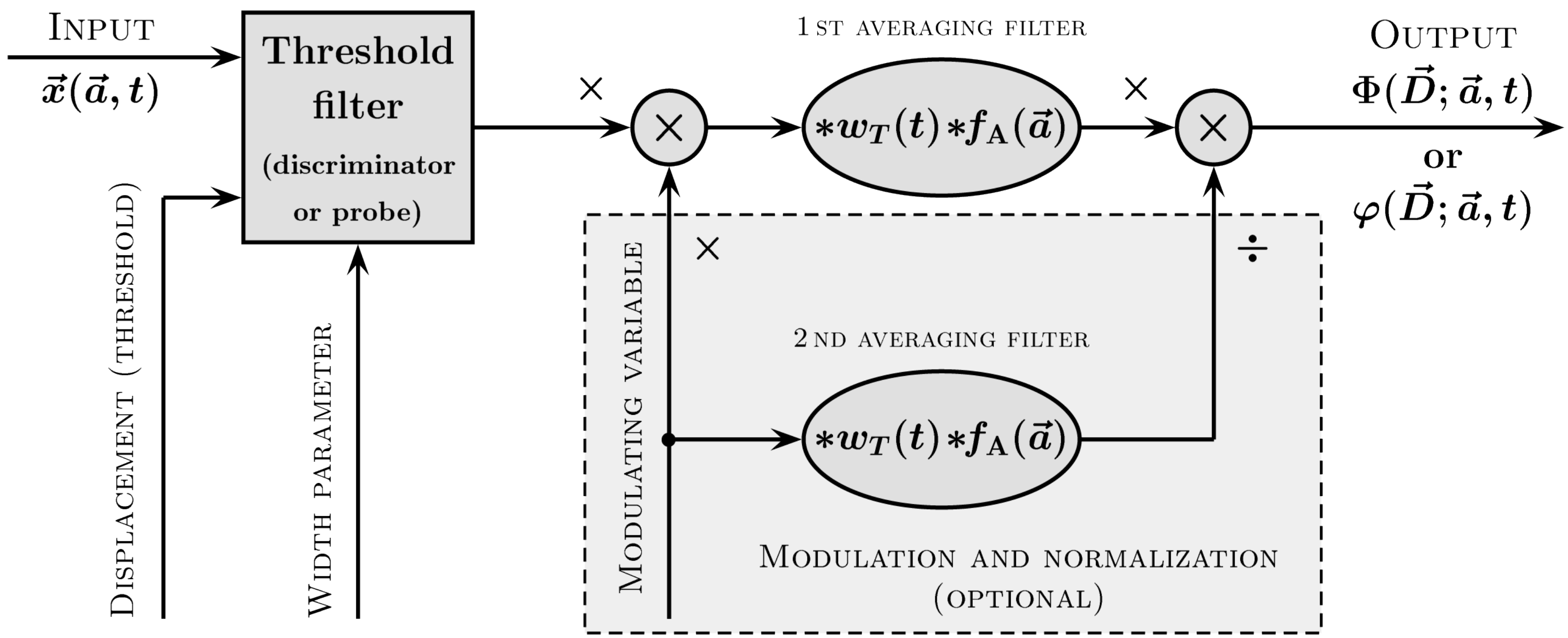
- Various physical measurements deal with rates of occurrence of different features of a signal. These features can be viewed as discrete coincidence events, e.g.:
  - Crossings of ***x*(*t*)** with a given threshold ***D***
  - Occurrence of extrema of ***x*(*t*)** of certain amplitude(s)
  - Various other conditional outcomes
- Example:
  - The rate **R** of crossings of ***D*** by a scalar signal ***x*(*t*),** measured by an ideal discriminator can be written as

$$\mathcal{R}(D,t) = w(t) * \{|\dot{x}(t)|\,\delta\,[D - x(t)]\}$$

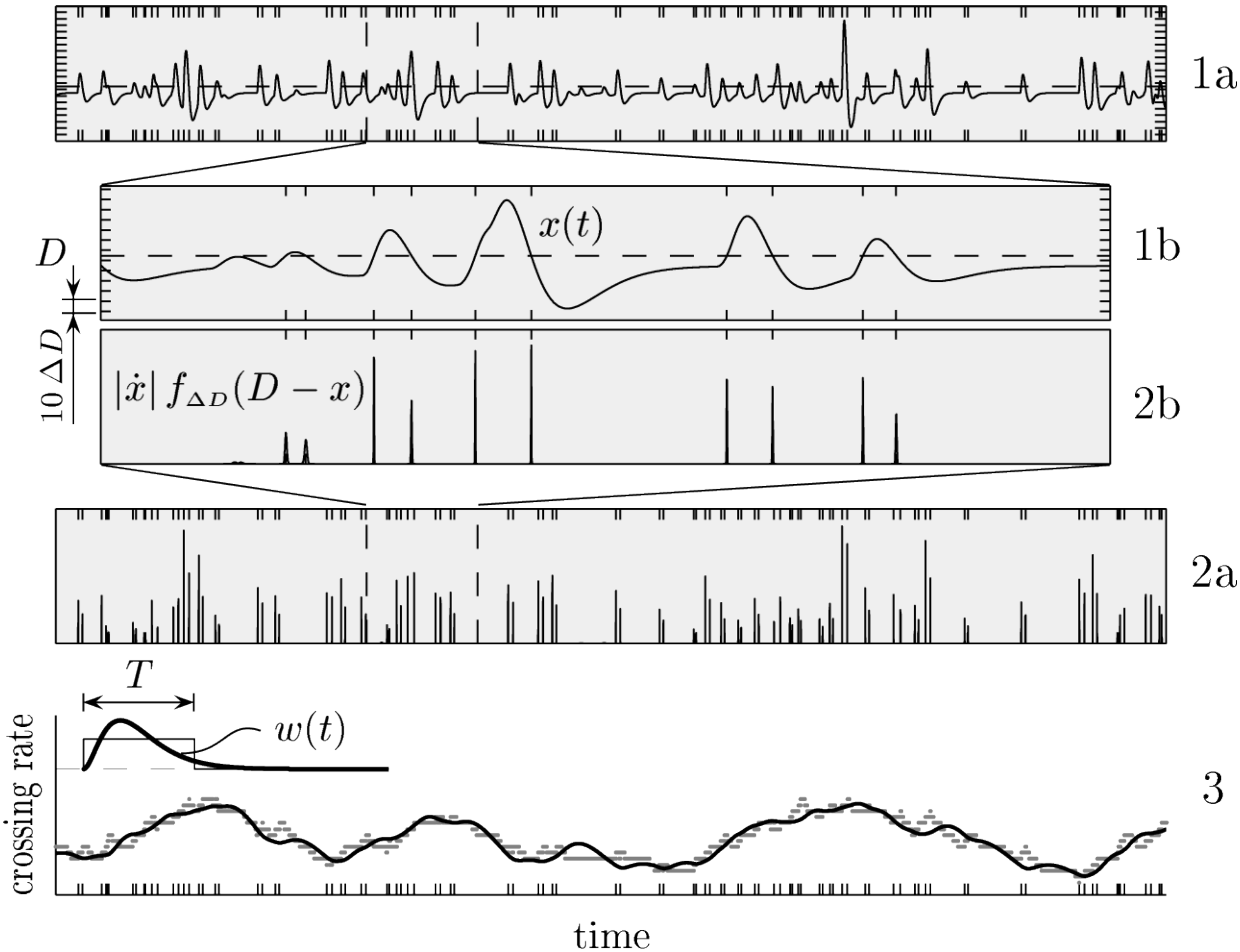
  - The rate measured by a real discriminator $\mathbf{F}_{\Delta \mathbf{D}}$ is

$$\mathcal{R}(D,t) = w(t) * \{|\dot{x}(t)|\,f_{\Delta D}\,[D - x(t)]\}$$

Basic generic measuring module



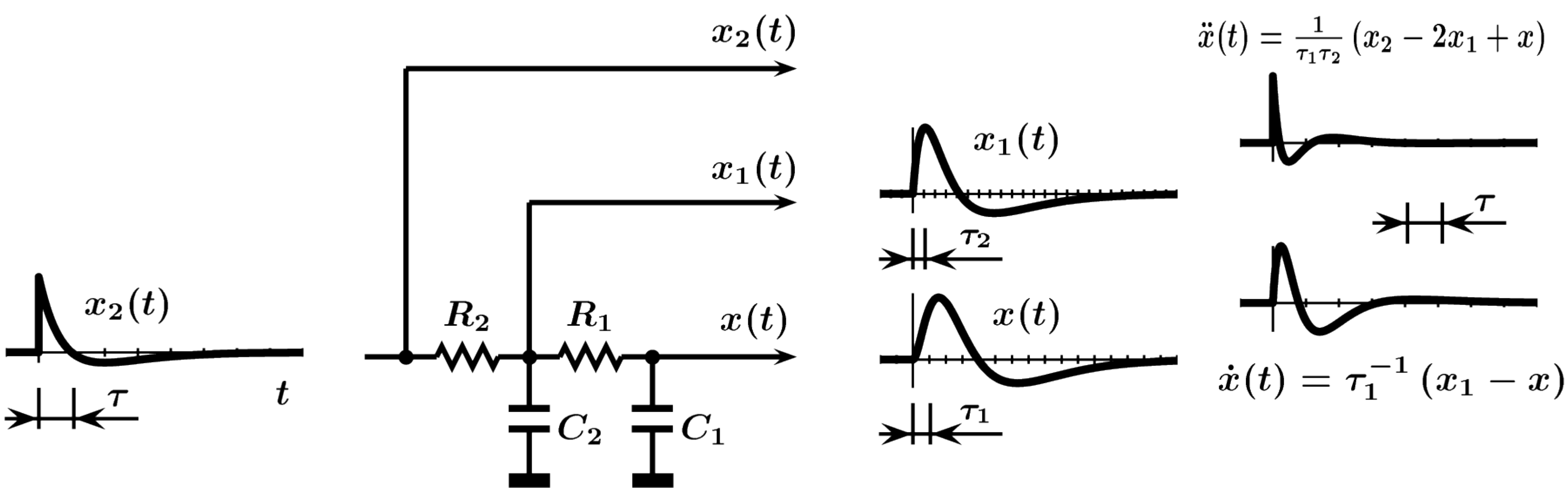
Measuring rates of crossings of signal ***x(t)*** with threshold ***D*** by a fast real discriminator



Derivatives of ***x*(*t*)** do not pose a problem in the analog domain

- Physical sensors have continuous time responses (typically exponential)
- Output signal is a convolution of the input with these responses
- Intermediate signals are available before and after some stage(s) of integration
- Derivatives can be obtained as linear combination of the intermediate signals

Obtaining time derivatives of the output signal ***x*(*t*)** as the real time difference between intermediate signals



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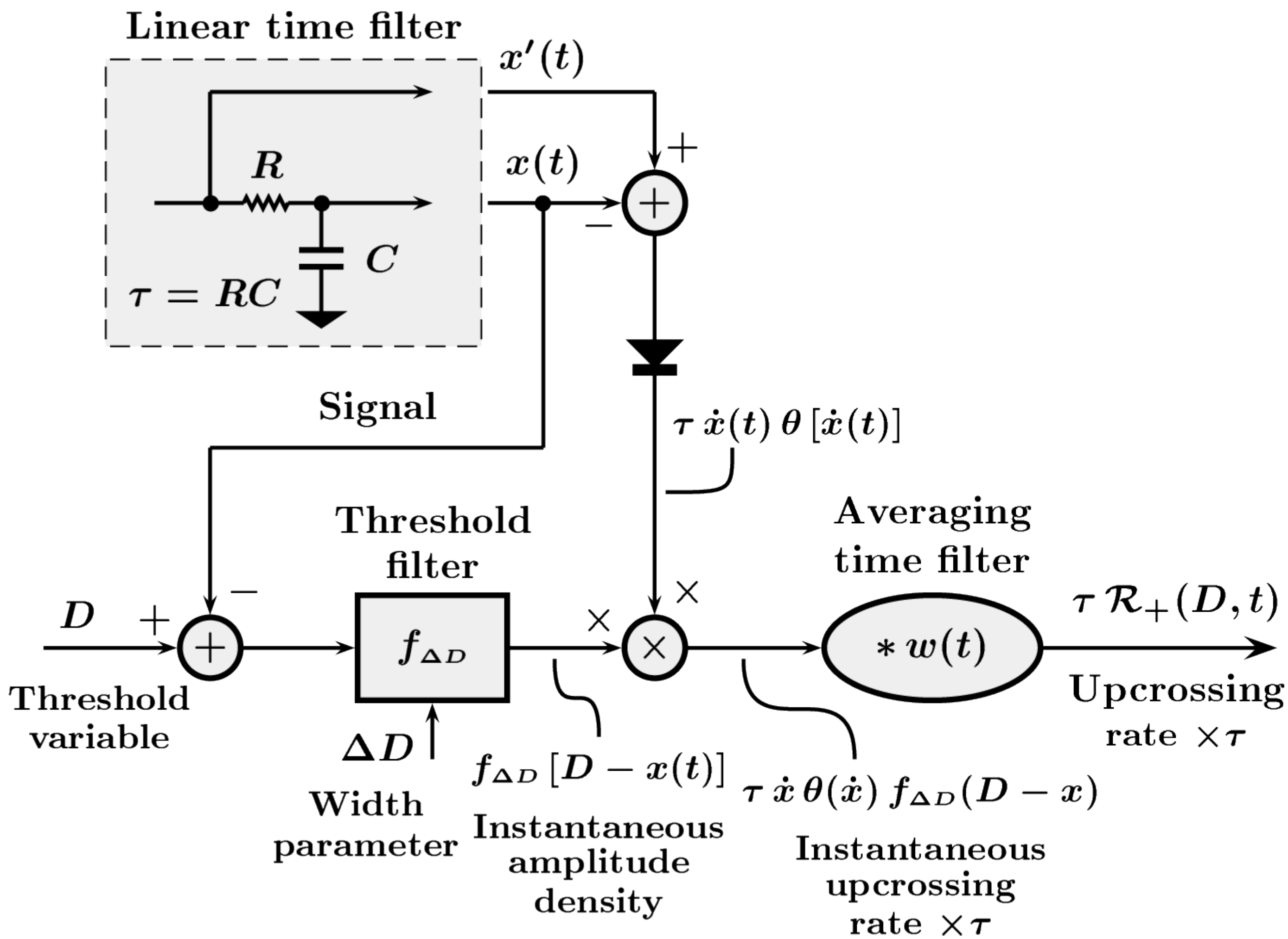
Simplified schematic of a positive slope threshold crossing counter



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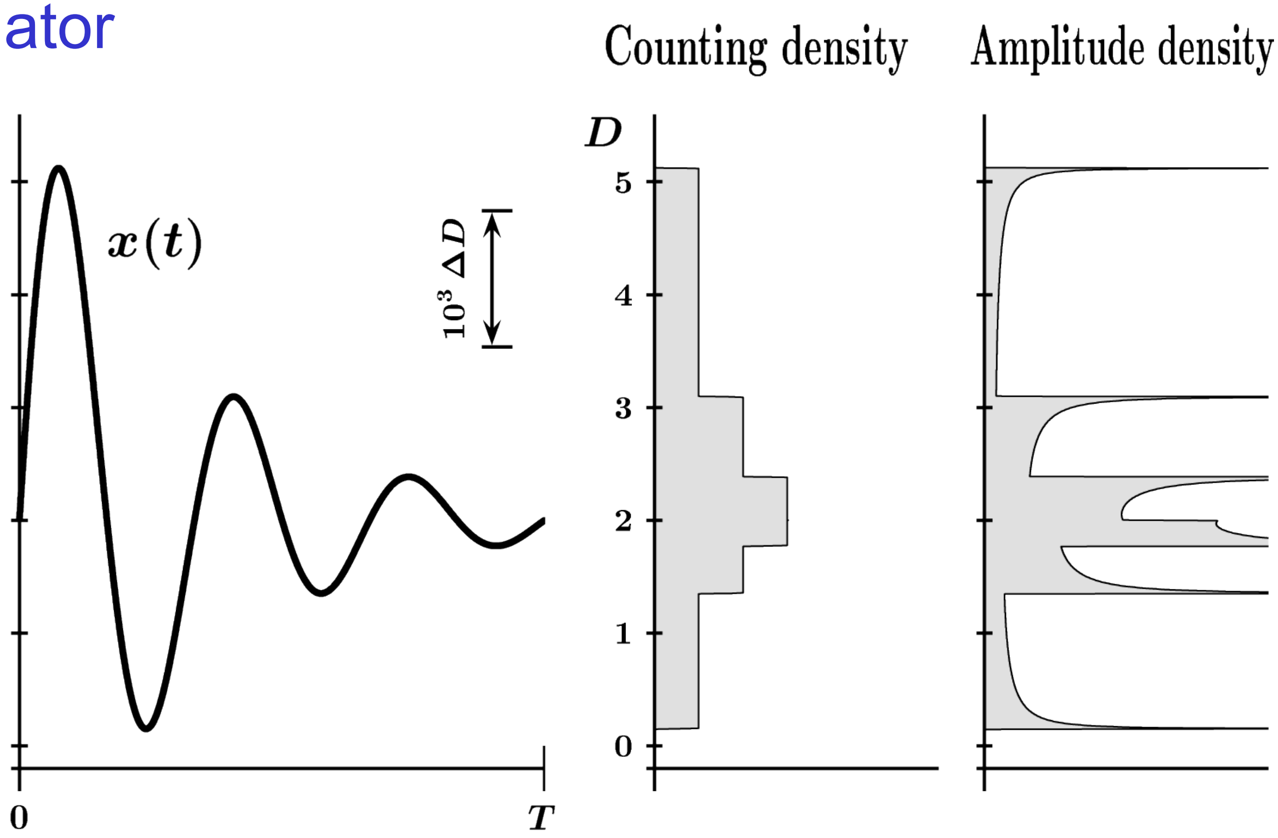
Modulated threshold density

$$\varphi(D,t) = \frac{w(t) * \{K(t)\, f_{\Delta D}\,[D - x(t)]\}}{w(t) * K(t)},$$

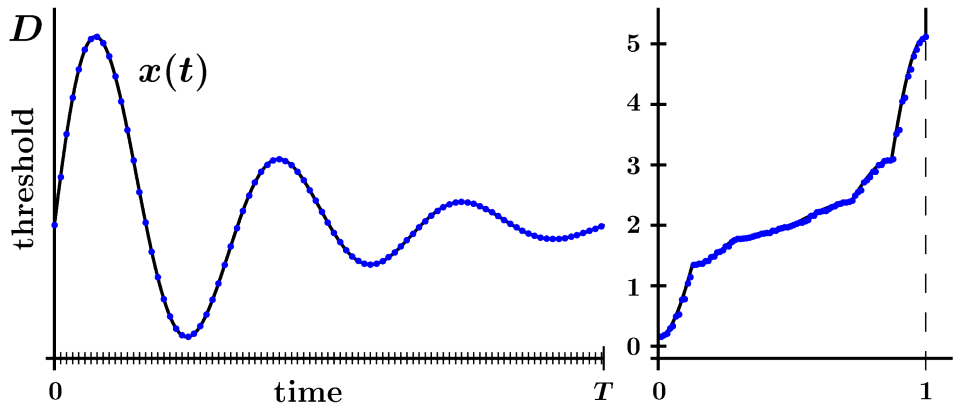
where $K(t)$ is a unipolar modulating signal

- Amplitude ($\boldsymbol{K(t)}$ **= const.**) and counting ($\boldsymbol{K(t) = |\mathrm{d}x/\mathrm{d}t|}$) densities for the fragment of a signal from a damped oscillator

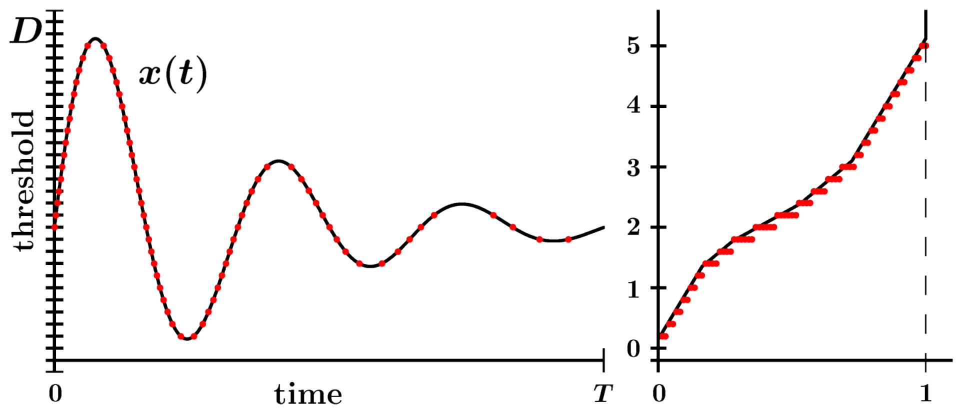


How do amplitude and counting distributions of a continuous signal relate to the distribution of a digitally sampled signal?

- Amplitude distribution relates to the distribution of a time-sampled signal



- Counting distribution relates to the distribution of a threshold-sampled signal



Amplitude and counting densities for vector signals

can be measured by a probe

$$f_{\mathrm{R}}(\vec{D}) \geq 0\,, \quad \int_{-\infty}^{\infty} \mathrm{d}^n\mathrm{r}\; f_{\mathrm{R}}(\vec{r}) = 1$$

- Amplitude density $\varphi(\vec{D}, t) = w(t) * f_{\mathrm{R}}\left[\vec{D} - x(t)\right]$
  - Characterizes time the signal spends in a vicinity of a certain point in the threshold space
- Counting density

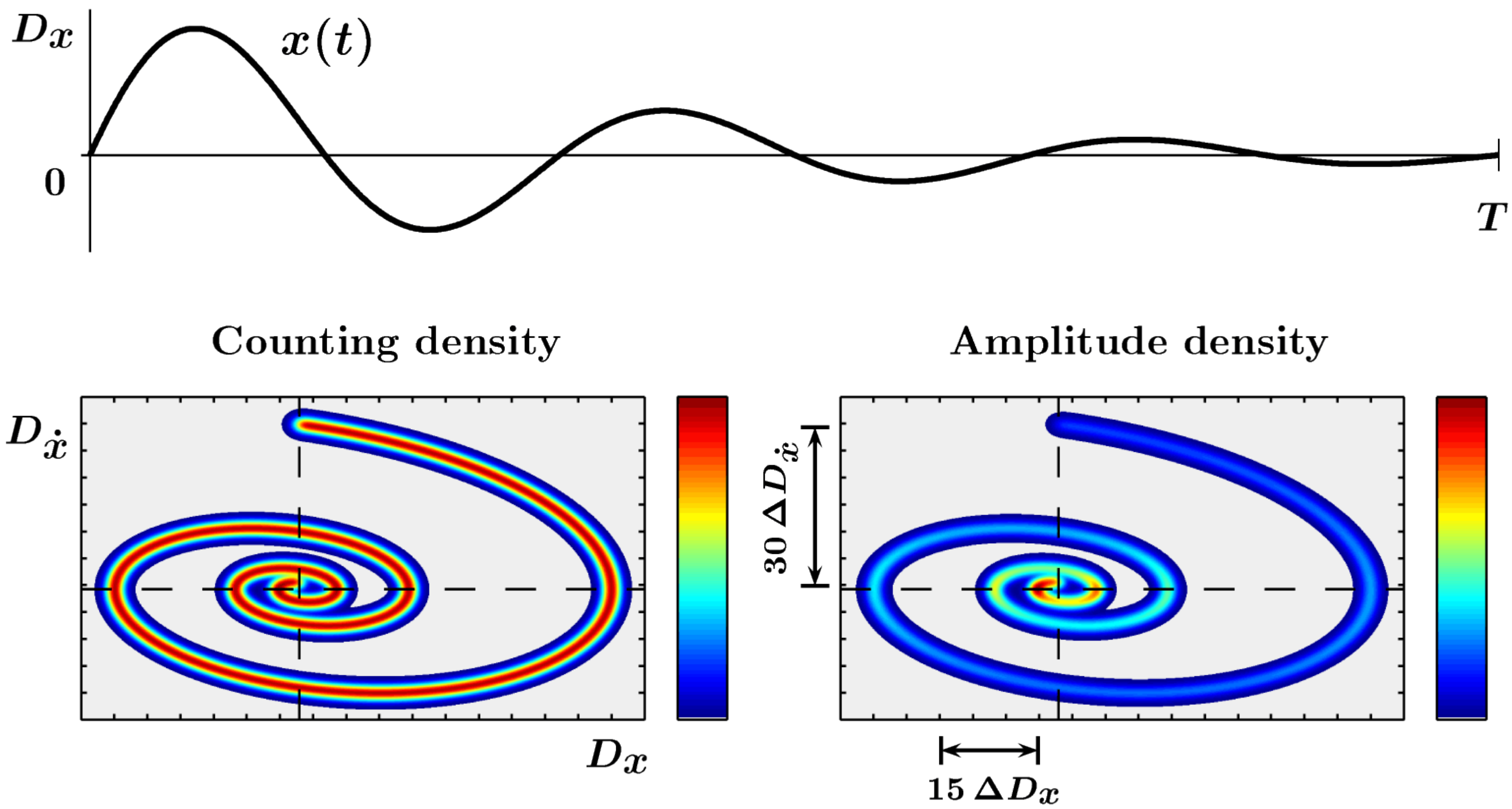
$$\phi(\vec{D}, t) = \frac{w(t) * \left\{|\dot{\vec{x}}(t)|\, f_{\mathrm{R}}\left[\vec{D} - x(t)\right]\right\}}{w(t) * |\dot{\vec{x}}(t)|}$$

  - Characterizes frequency of 'visits' to this vicinity by the signal
  - Numerator is the counting rate

$$\mathcal{R}(\vec{D}, t) = w(t) * \left\{|\dot{\vec{x}}(t)|\, f_{\mathrm{R}}\left[\vec{D} - x(t)\right]\right\}$$

  - $|\dot{\vec{x}}(t)| = \sqrt{\sum_{i=1}^{n}\left[\frac{\dot{x}_i(t)}{\Delta D_i}\right]^2}$

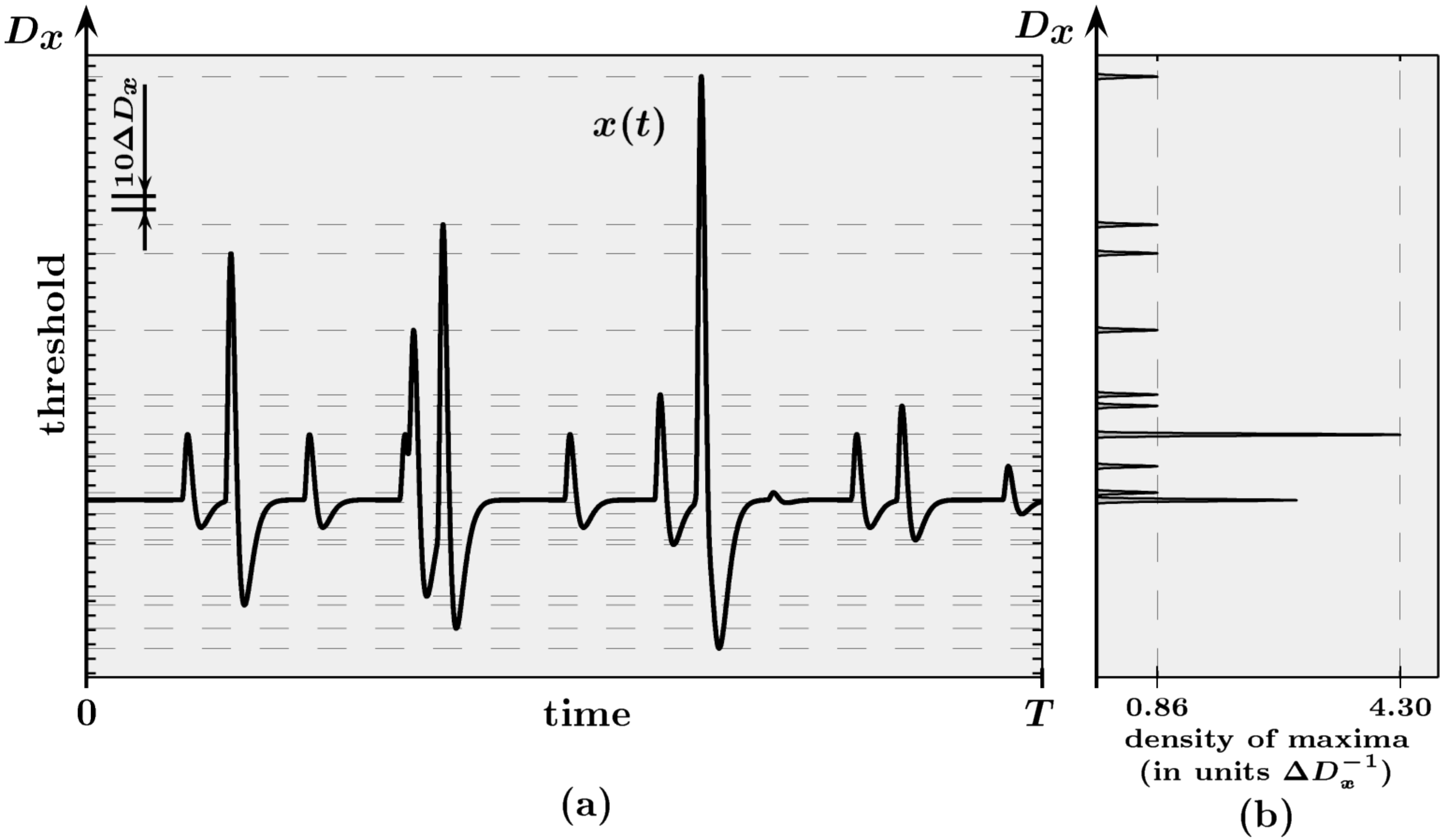
Amplitude and counting densities for the fragment of a signal from a damped oscillator



Counting maxima in a signal

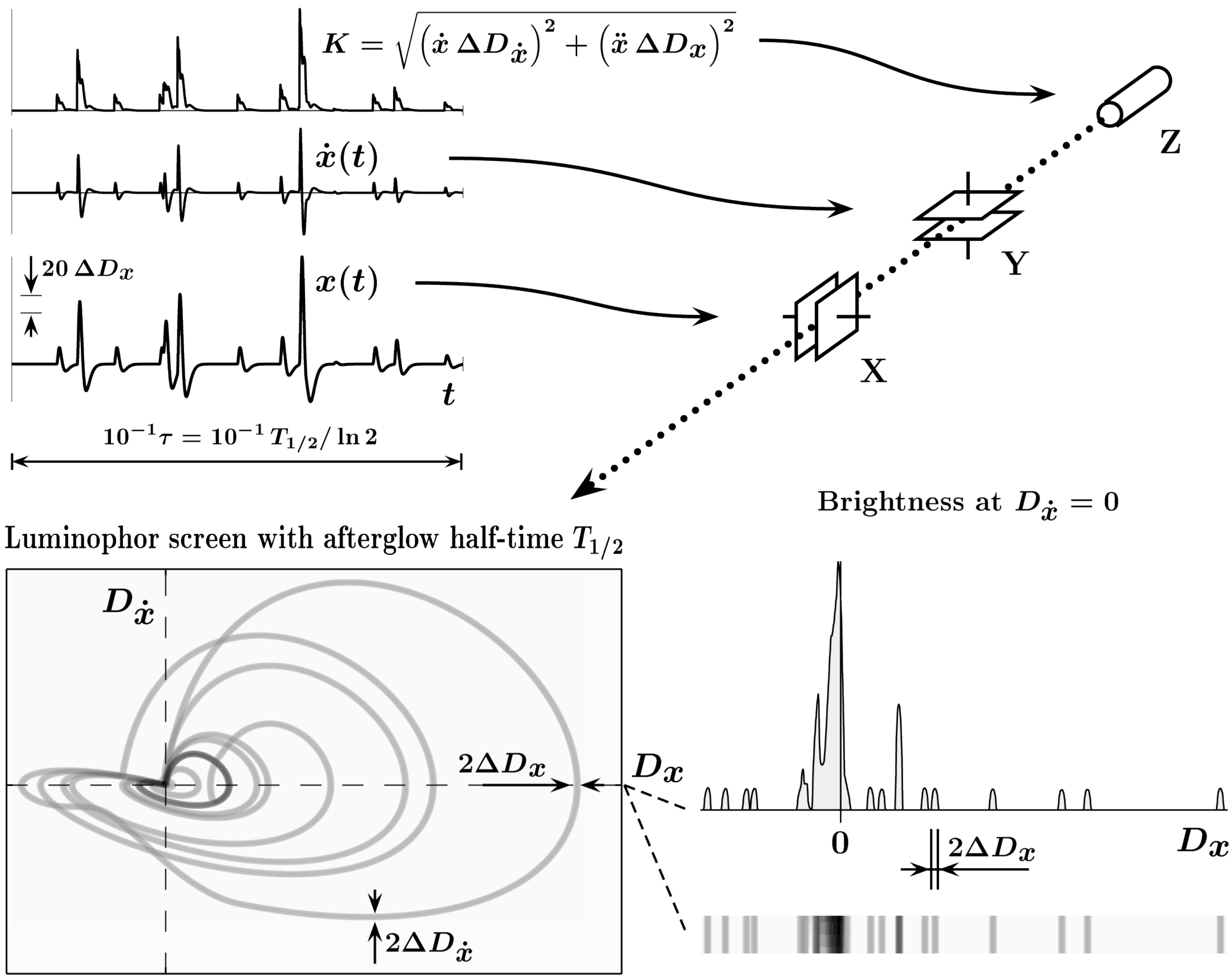
Panel (a): Fragment of a signal in the interval [0, T]

Panel (b): Density of maxima

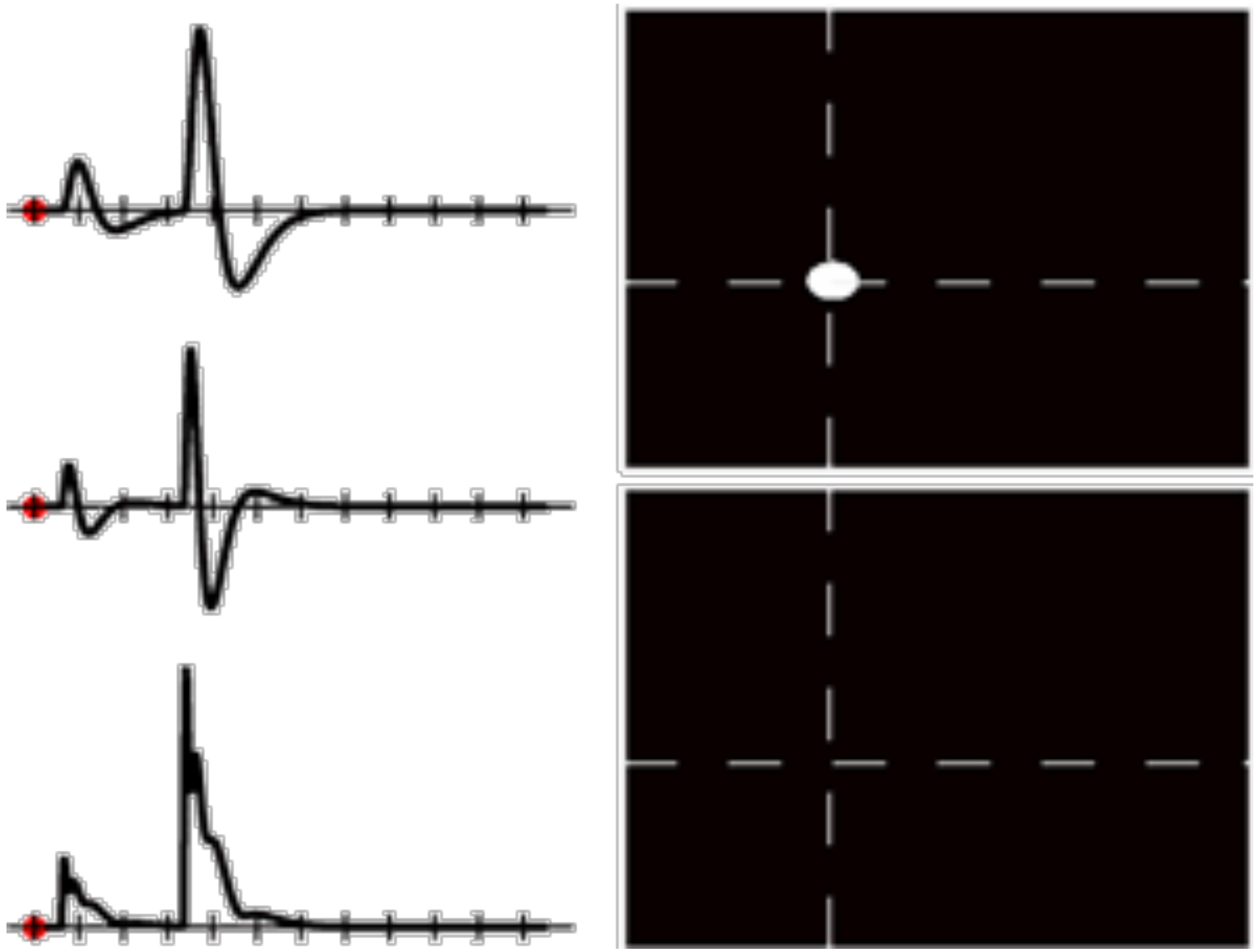


Multivariate counting measurements

Using conventional analog oscilloscope for counting signal's stationary points



Using analog oscilloscope for counting signal's stationary points



ANIMATED

Real-time entropy-like measurements

Basis for real-time entropy-like measurements

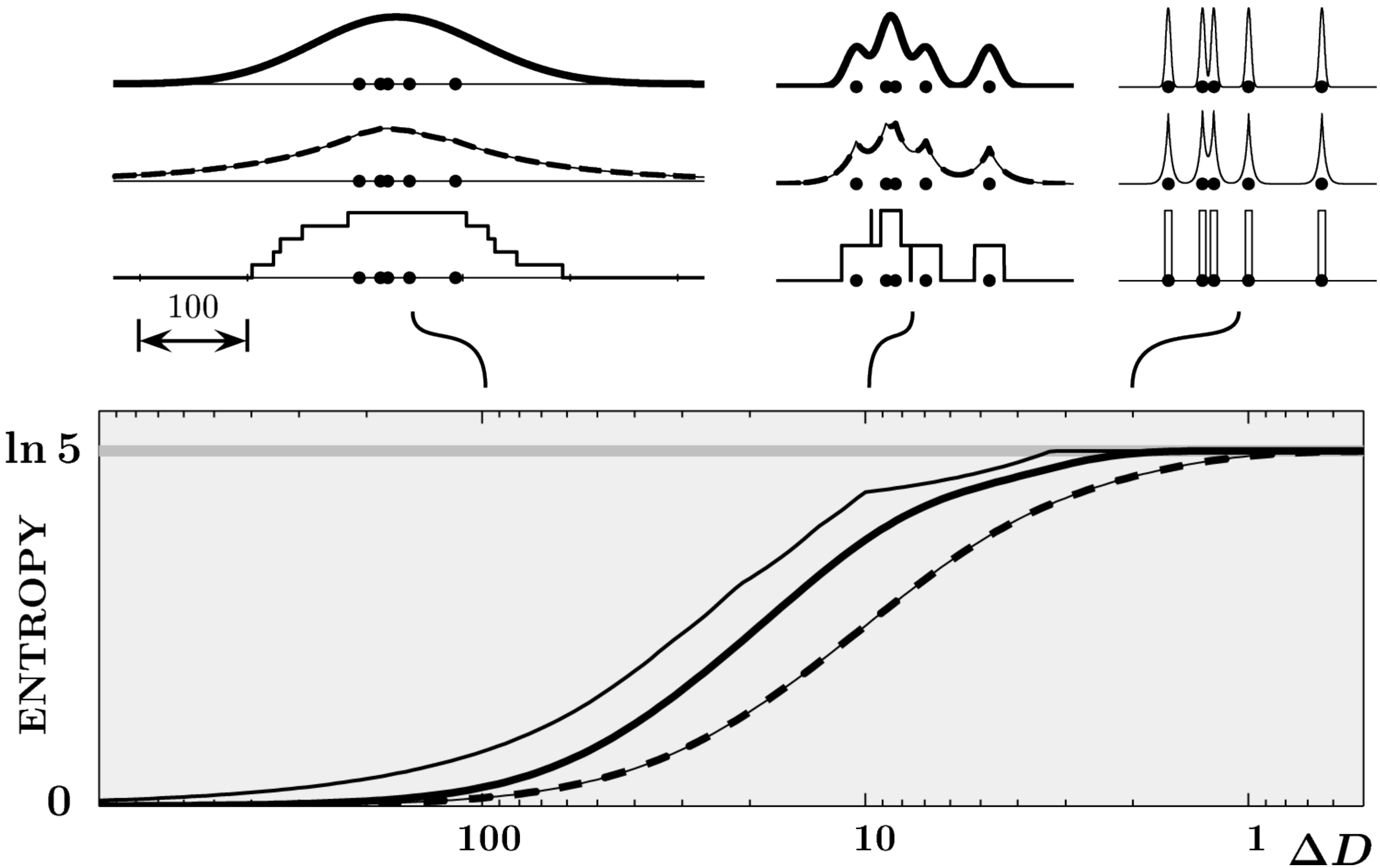
We can define the (time dependent) entropy $\mathcal{H}(t)$ for the density $\varphi(\vec{D},t)$ of a signal $\vec{x}(t)$ as

$$\mathcal{H}(t) = C_f - \int_{-\infty}^{\infty} \mathrm{d}r\, \varphi(\vec{r},t) \ln\left[\frac{\varphi(\vec{r},t)}{f_{\mathrm{R}}(0)}\right] \geq 0\,,$$

where $f_{\mathrm{R}}(0)$ is the *modal value* of f_{R}, and C_f is a constant property of the probe (dependent only on the *shape* of f_{R})

$$C_f = \int_{-\infty}^{\infty} \mathrm{d}\alpha\, f_{\mathrm{R}}(\vec{\alpha}) \ln\left[\frac{f_{\mathrm{R}}(\vec{\alpha})}{f_{\mathrm{R}}(0)}\right] \leq 0\,.$$

- $\boldsymbol{f_R(0)}$ is the maximum possible value of the density $\boldsymbol{f}$ we can get from our measurements by the probe $\boldsymbol{f_R}$
- $\boldsymbol{f_R(0)^{-1}}$ is the *elemental phase volume* of the threshold space


100
ln 5
ENTROPY
0
100
10
1
ΔD

Summary

- Consideration of finite precision and inertial properties of data acquisition systems allows us to model measurements by ‘slow real discriminators’
- Various signal processing tasks can be formulated in terms of continuous time dependent distribution and density functions
- Analysis through analog representation allows simple and efficient implementation of traditionally digital-only techniques, and the introduction of new signal characteristics. Examples include:
  - Nonlinear filtering techniques based on order statistics
  - Multivariate counting measurements
  - Real-time entropy-like measurements

AvaTekh LLC

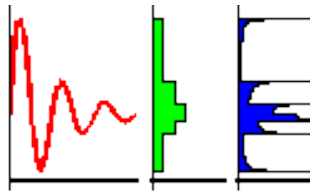
Robust analog approach to data acquisition and analysis

AVATEKH = AVAtar + TEKHnē

AVATAR = Analysis of VAriables Through Analog Representation

Greek *TEKHNĒ* = art, skill

AVATAR



Analysis of VAriables Through Analog Representation

"– while linkage to macroscopic machinery has not proven cost-effective, the case has turned out to be otherwise for monitoring and controlling scientific instruments. For this it is inadequate to supply the operating brain with numbers such as voltmeter reading and nothing else. For example, a spectrum is best considered–rationally appreciated–when the operator sees it and, simultaneously, knows the exact wavelength and intensity of every line. Through appropriate hardware and software, this can now be done." — From "The Avatar" by Poul Anderson

Backup slides
(no handouts)

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$$\delta[a - f(x)] = \sum_i \frac{\delta(x - x_i)}{|f'(x_i)|}$$

- $|f^{0}(x_i)|$ is the absolute value of the derivative of $f(x)$ at x_i
- sum goes over all x_i such that $f(x_i) = a$

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Boxcar probe ***b_a***

in the equation for *a*-trimmed mean filter

$$b_\alpha(x) = \frac{1}{1-2\alpha}\left[\theta(x-\alpha) - \theta(x-1+\alpha)\right]$$

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Threshold distribution / density measured by a slow real discriminator / probe with hysteresis

- Distribution Φ**(*D*,*t*):**

$$\Phi(D,t) = w(t) * \Big\{ \theta[\dot{x}(t)] \mathcal{F}_{\Delta D}[D - x(t) - \delta D] + \theta[-\dot{x}(t)] \mathcal{F}_{\Delta D}[D - x(t) + \delta D] \Big\}$$

- Density φ**(*D*,*t*):**

$$\varphi(D,t) = w(t) * \Big\{ \theta[\dot{x}(t)] f_{\Delta D}[D - x(t) - \delta D] + \theta[-\dot{x}(t)] f_{\Delta D}[D - x(t) + \delta D] \Big\}$$

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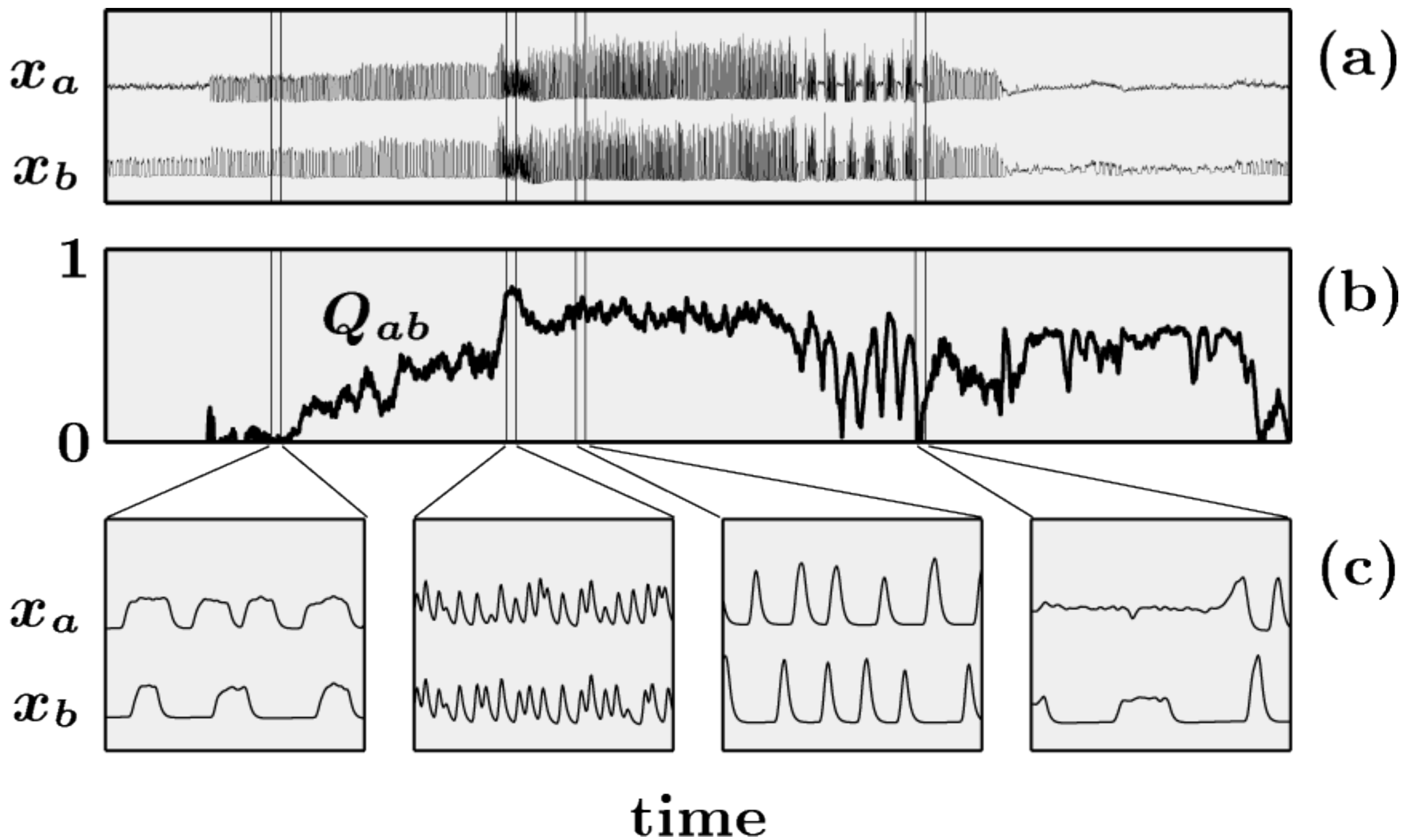
Quantification of “visual” similarity between two signals through overlapping of their respective quantile domains



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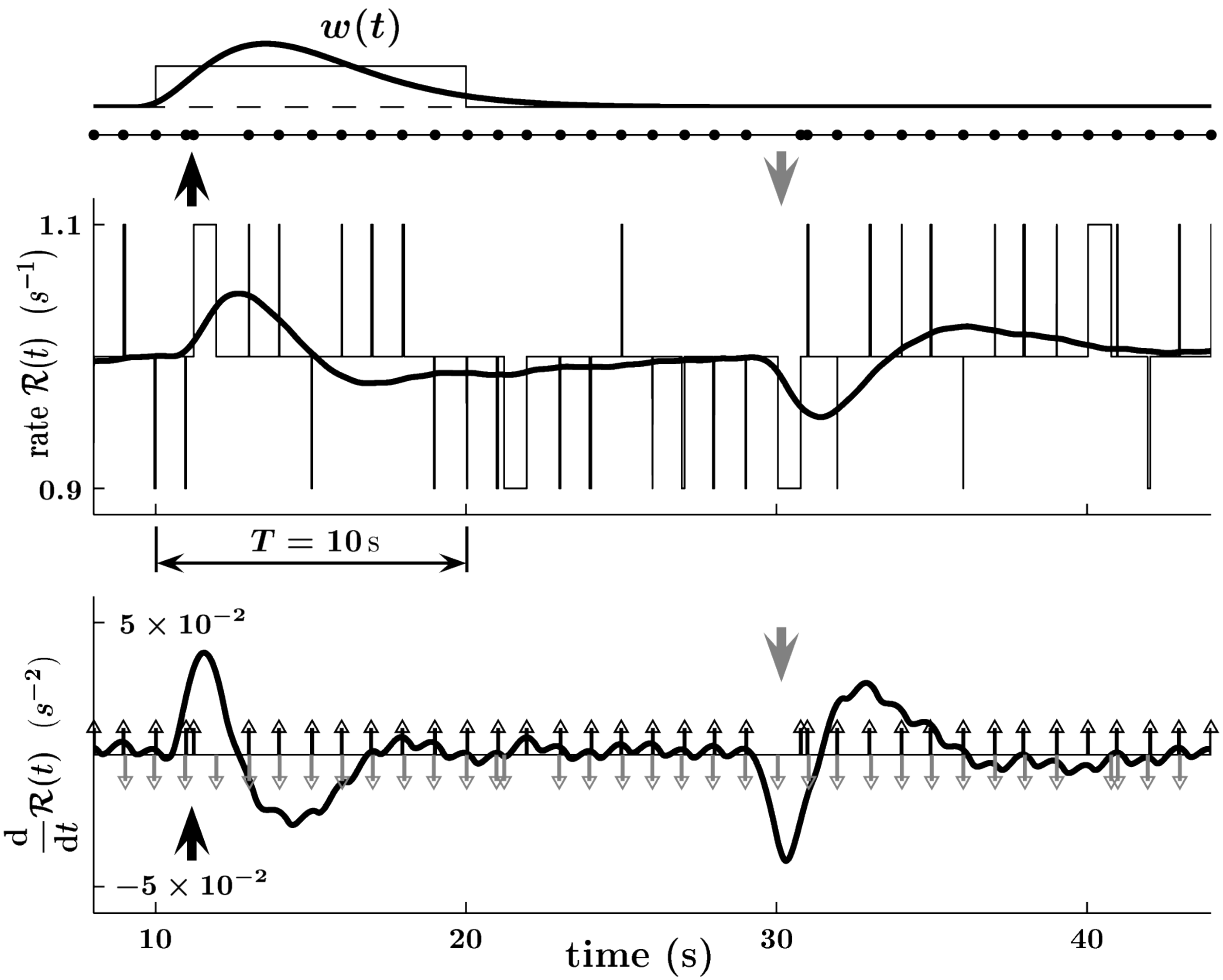
Using differentiability of rate measured with a continuous test function for detection and quantification of arrhythmia



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Space science instrumentation

- Onboard / *in situ* science capabilities
- No software / firmware requirements
- Flexible models for linking observables to quantities of interest
- Automated adaptive data acquisition
- Quantitative treatment of uncertainty present in data
- Effective organization of data for transmission and storage

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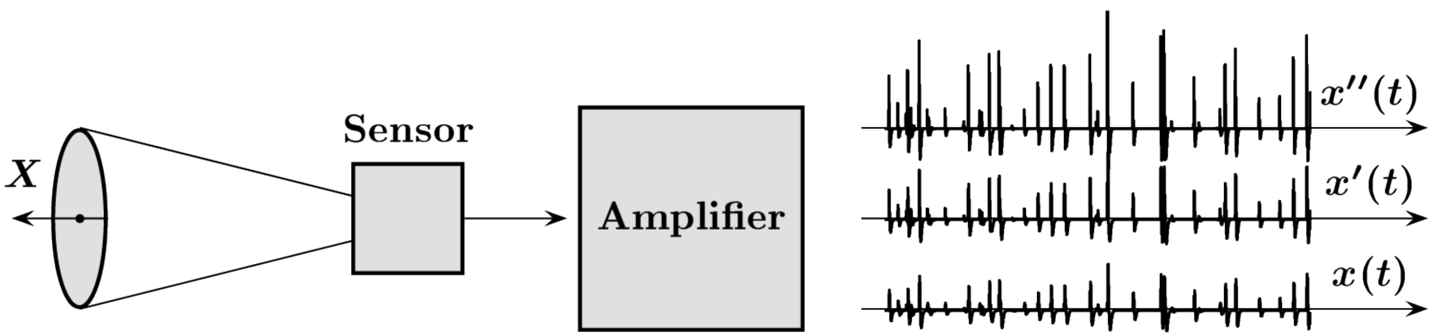
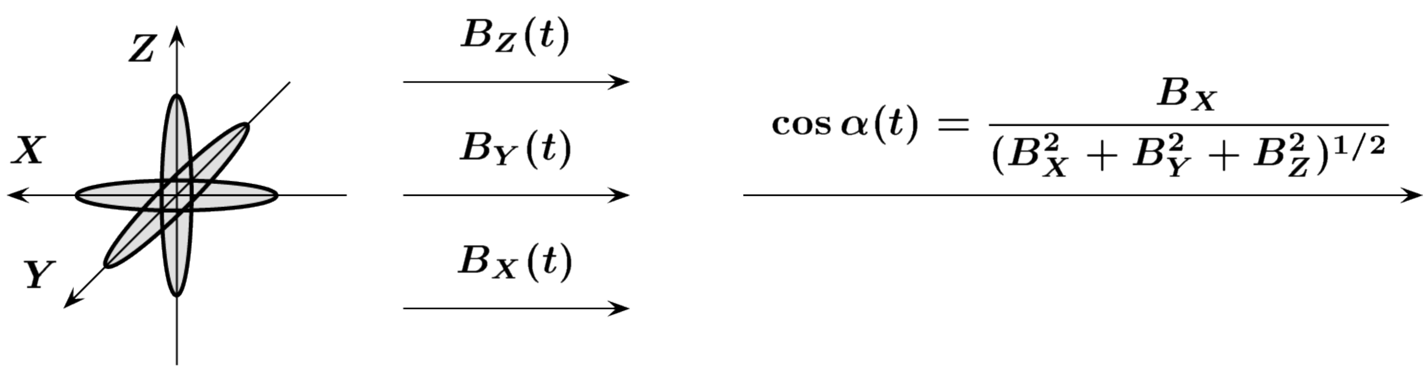
Example I: Integrated energy - pitch angle measurements

Particle detector



Magnetometer



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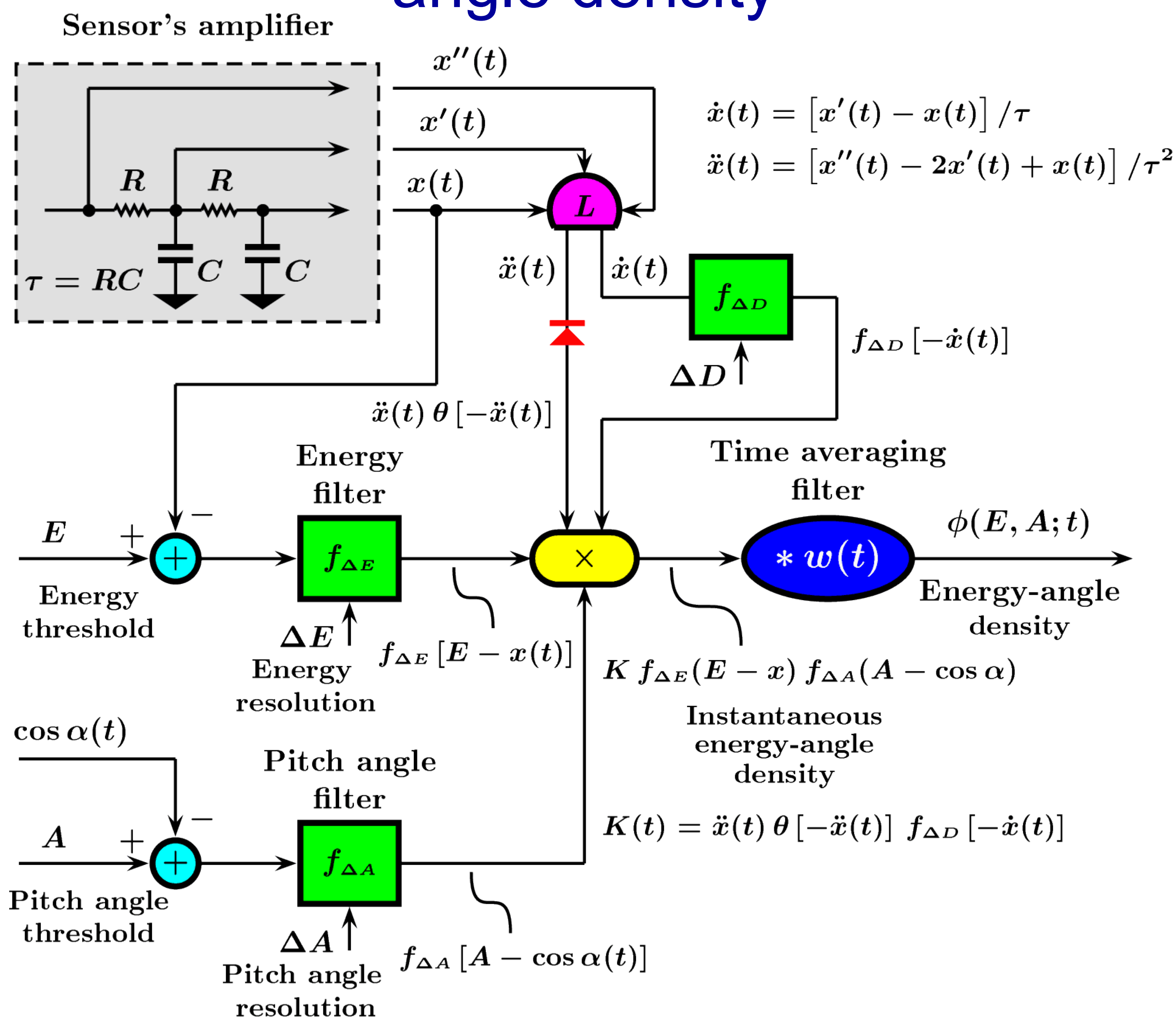
Direct analog measurement of energy - pitch angle density



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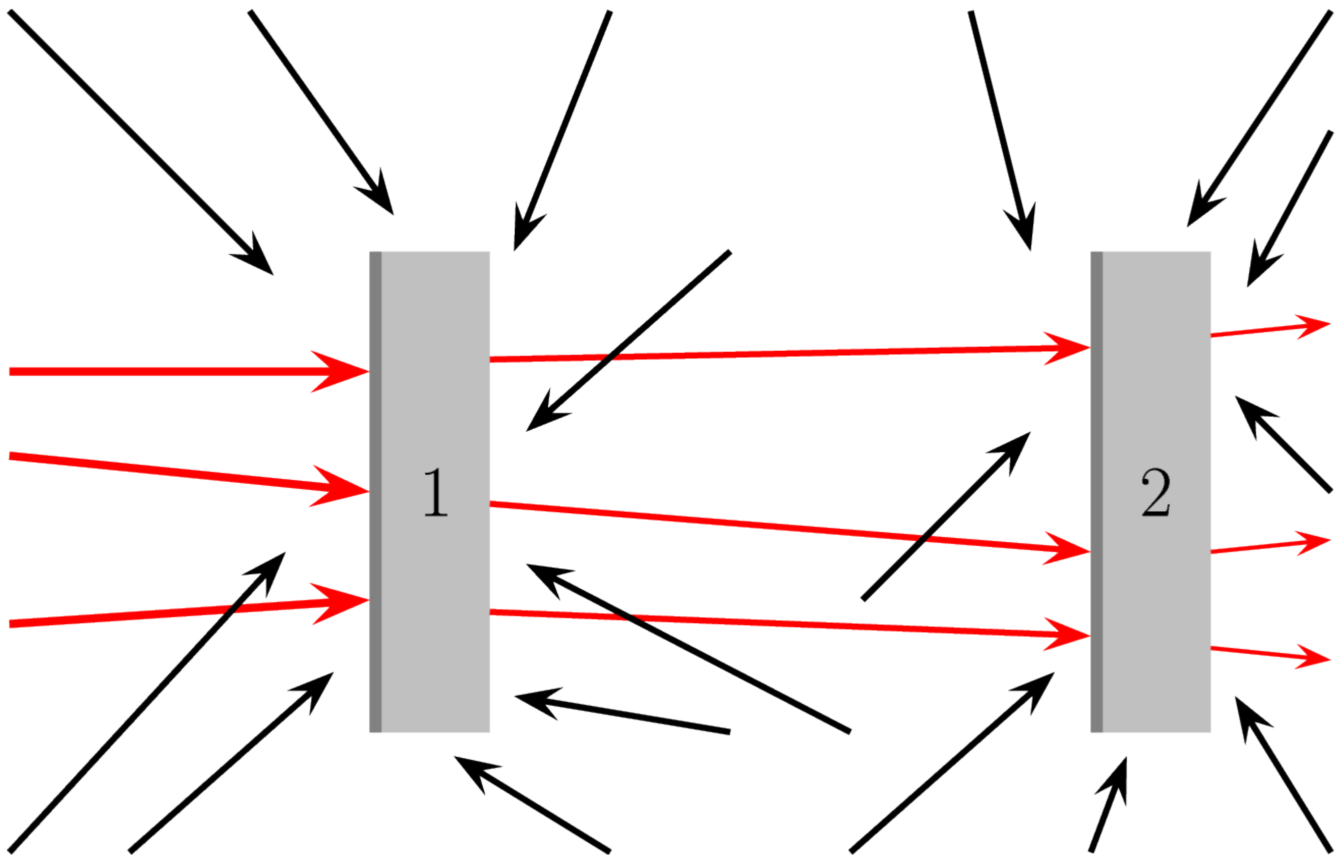
Example II: Directional particle flux measurements

Suppression of omnidirectional flux by analog coincidence counting



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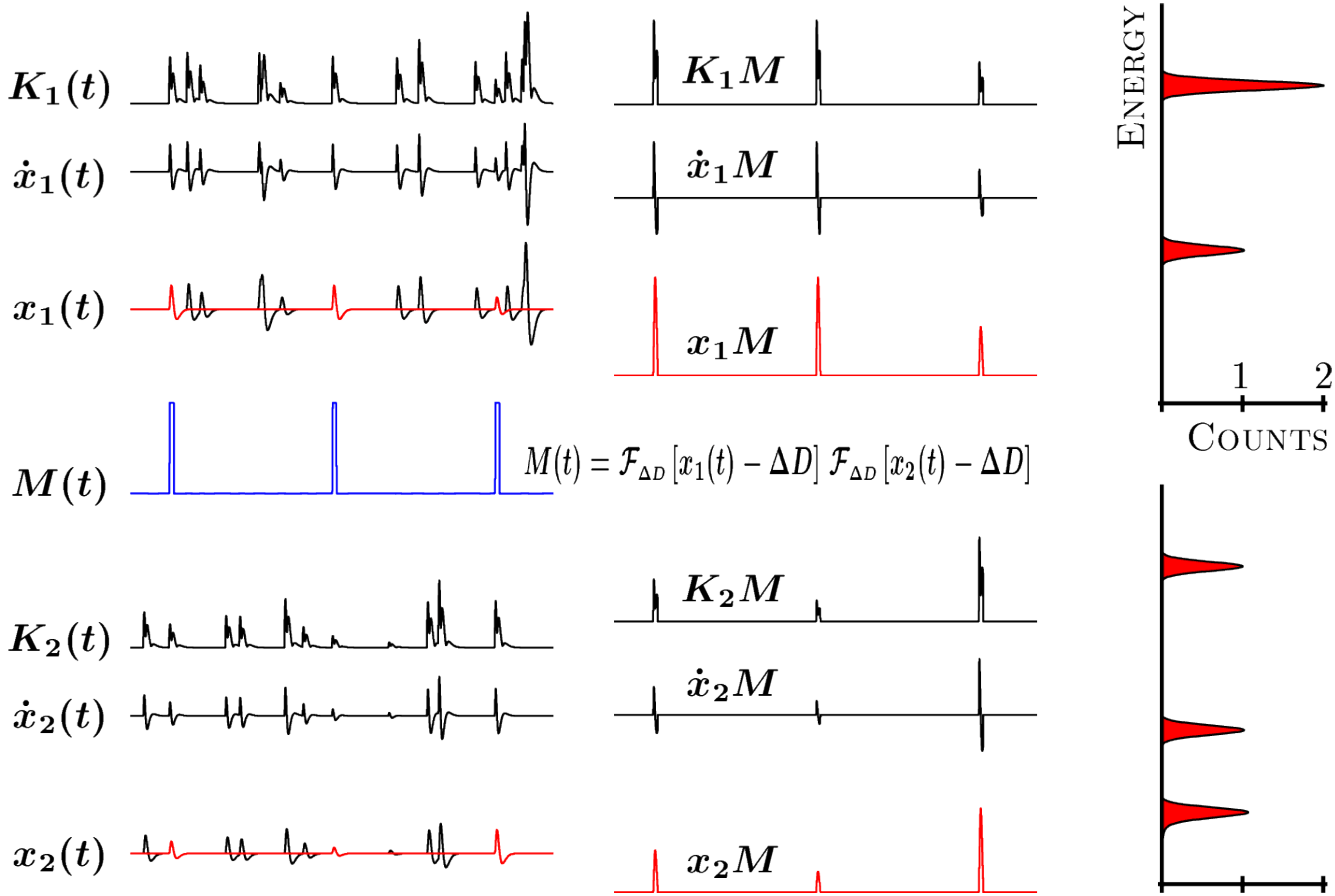
Counting synchronous pulses (shown in red) from two detectors



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